Section P.1
Algebraic Expressions, Mathematical Models, and Real Numbers

It costs how much?

You are looking ahead to the next school year and wondering how much money you will need.
Is there any way that you can use trends for college costs over the past few years to predict how much college will cost next year?
In the Exercise Set for this section, you will use a model that will allow you to project average costs at private U.S. colleges in the near future.

Objective #1: Evaluate algebraic expressions.

✔ *Solved Problem #1*

1. Evaluate $8+6(x-3)^2$ for $x = 13$.

$$\begin{aligned} 8+6(x-3)^2 &= 8+6(13-3)^2 \\ &= 8+6(10)^2 \\ &= 8+6(100) \\ &= 8+600 \\ &= 608 \end{aligned}$$

✎ *Pencil Problem #1* ✎

1. Evaluate $4+5(x-7)^3$ for $x = 9$.

Objective #2: Use mathematical models.

✔ *Solved Problem #2*

2. The formula $T = 4x^2 + 341x + 3194$ models the average cost of tuition and fees, T, for public U.S. colleges for the school year ending x years after 2000. Use this formula to project the average cost of tuition and fees at public U.S. colleges for the school year ending in 2015.

Because 2015 is 15 years after 2000, we substitute 15 for x in the formula.

$$T = 4x^2 + 341x + 3194$$
$$T = 4(15)^2 + 341(15) + 3194$$
$$T = 4(225)^2 + 341(15) + 3194$$
$$T = 900 + 5115 + 3194$$
$$T = 9209$$

The formula indicates that for the school year ending in 2015, the average cost of tuition and fees at public U.S. colleges will be $9209.

✎ *Pencil Problem #2* ✎

2. The formula $T = 26x^2 + 819x + 15{,}527$ models the average cost of tuition and fees, T, for private U.S. colleges for the school year ending x years after 2000. Use this formula to project the average cost of tuition and fees at private U.S. colleges for the school year ending in 2013.

Objective #3: Find the intersection of two sets.

✔ *Solved Problem #3*

3. Find the intersection: $\{3, 4, 5, 6, 7\} \cap \{3, 7, 8, 9\}$.

The elements common to $\{3, 4, 5, 6, 7\}$ and $\{3, 7, 8, 9\}$ are 3 and 7.

$\{3, 4, 5, 6, 7\} \cap \{3, 7, 8, 9\} = \{3, 7\}$

✎ *Pencil Problem #3* ✐

3. Find the intersection: $\{1, 2, 3, 4\} \cap \{2, 4, 5\}$.

Objective #4: Find the union of two sets.

✔ *Solved Problem #4*

4. Find the union: $\{3, 4, 5, 6, 7\} \cup \{3, 7, 8, 9\}$.

List the elements from the first set: 3, 4, 5, 6, and 7. Now list any elements from the second set not in the first: 8 and 9.

$\{3, 4, 5, 6, 7\} \cup \{3, 7, 8, 9\} = \{3, 4, 5, 6, 7, 8, 9\}$

✎ *Pencil Problem #4* ✐

4. Find the union: $\{1, 2, 3, 4\} \cup \{2, 4, 5\}$.

Objective #5: Recognize subsets of the real numbers..

✔ *Solved Problem #5*

5. Consider the following set of numbers:
$\left\{-9, -1.3, 0, 0.\overline{3}, \frac{\pi}{2}, \sqrt{9}, \sqrt{10}\right\}$.

5a. List the natural numbers.

The natural numbers are used for counting. The only natural number is $\sqrt{9}$ because $\sqrt{9} = 3$.

5b. List the rational numbers.

All numbers that can be expressed as quotients of integers are rational numbers: $-9\left(-9 = \frac{-9}{1}\right)$, $0\left(0 = \frac{0}{1}\right)$, and $\sqrt{9}\ \left(\sqrt{9} = \frac{3}{1}\right)$. All numbers that are terminating or repeating decimals are rational numbers: -1.3 and $0.\overline{3}$.

✎ *Pencil Problem #5* ✐

5. Consider the following set of numbers:
$\left\{-11, -\frac{5}{6}, 0, 0.75, \sqrt{5}, \pi, \sqrt{64}\right\}$.

5a. List the natural numbers.

5b. List the rational numbers.

Objective #6: Use inequality symbols.

✔ *Solved Problem #6*

6. Indicate whether each statement is true or false.

6a. $-8 > -3$

This statement is false. Because −8 lies to the left of −3 on a number line, −8 is less than −3. So, $-8 < -3$.

6b. $9 \le 9$

This statement is true because $9 = 9$.

✎ *Pencil Problem #6* ✎

6. Indicate whether each statement is true or false.

6a. $-7 < -2$

6b. $-5 \ge 2$

Objective #7: Evaluate absolute value.

✔ *Solved Problem #7*

7. Rewrite each expression without absolute value bars.

7a. $|1-\sqrt{2}|$

Because $\sqrt{2} \approx 1.4,$ the number $1-\sqrt{2}$ is negative.
Thus, $|1-\sqrt{2}| = -(1-\sqrt{2}) = \sqrt{2}-1.$

7b. $|\pi - 3|$

Because $\pi \approx 3.14,$ the number $\pi - 3$ is positive.
Thus, $|\pi - 3| = \pi - 3.$

7c. $\dfrac{|x|}{x}$ if $x > 0$

If $x > 0$, then $|x| = x$. Thus, $\dfrac{|x|}{x} = \dfrac{x}{x} = 1.$

✎ *Pencil Problem #7* ✎

7. Rewrite each expression without absolute value bars.

7a. $|12 - \pi|$

7b. $|\sqrt{2} - 5|$

7c. $\dfrac{-3}{|-3|}$

Objective #8: Use absolute value to express distance.

✔ *Solved Problem #8*

8. Find the distance between –4 and 5 on the real number line.

$|-4-5| = |-9| = 9$

✎ *Pencil Problem #8* ✎

8. Find the distance between −19 and –4 on the real number line.

Objective #9: Identify properties of the real numbers.

✔ *Solved Problem #9*

9. State the name of the property illustrated.

9a. $2+\sqrt{5}=\sqrt{5}+2$

The order of the numbers in the addition has changed. This illustrates the commutative property of addition.

9b. $1\cdot(x+3)=x+3$

One has been deleted from a product. This illustrates the identity property of multiplication.

✎ *Pencil Problem #9* ✐

9. State the name of the property illustrated.

9a. $6+(2+7)=(6+2)+7$

9b. $2(-8+6)=-16+12$

Objective #10: Simplify algebraic expressions.

✔ *Solved Problem #10*

10. Simplify: $6+4[7-(x-2)]$.

$$\begin{aligned} 6+4[7-(x-2)] &= 6+4[7-x+2] \\ &= 6+4[9-x] \\ &= 6+36-4x \\ &= (6+36)-4x \\ &= 42-4x \end{aligned}$$

✎ *Pencil Problem #10* ✐

10. Simplify: $7-4[3-(4y-5)]$.

<u>**Answers** for Pencil Problems *(Textbook Exercise references in parentheses)*</u>:

1. 44 *(P.1 #9)* **2.** \$30,568 *(P.1 #131c)* **3.** $\{2, 4\}$ *(P.1 #21)* **4.** $\{1, 2, 3, 4, 5\}$ *(P.1 #29)*
5. a. $\sqrt{64}$ **b.** $-11,\ -\frac{5}{6},\ 0,\ 0.75,\ \sqrt{64}$ *(P.1 #37)* **6. a.** true **b.** false **7. a.** $12-\pi$ *(P.1 #53)*
b. $5-\sqrt{2}$ *(P.1 #55)* **c.** -1 *(P.1 #57)* **8.** 15 *(P.1 #71)* **9. a.** associative property of addition *(P.1 #77)*
b. distributive property *(P.1 #81)* **10.** $16y-25$ *(P.1 #93)*

Section P.2
Exponents and Scientific Notation

WOW, THAT'S BIG!

Did you know that in the summer of 2012 the national debt passed \$16,000,000,000,000 or \$16 trillion? Yes, that's 12 zeros you count. In this section, you will express the national debt in a form called *scientific notation* and use this form to calculate your share of the debt.

Objective #1: Use the properties of exponents.

✔ ***Solved Problem #1***	✎ ***Pencil Problem #1***
1a. Multiply using the product rule: $3^3 \cdot 3^2$ $3^3 \cdot 3^2 = 3^{3+2} = 3^5$ or 243	**1a.** Multiply using the product rule: $x^3 \cdot x^7$
1b. Multiply using the product rule: $(4x^3y^4)(10x^2y^6)$ $(4x^3y^4)(10x^2y^6) = 4 \cdot 10 \cdot x^{3+2} \cdot y^{4+6} = 40x^5y^{10}$	**1b.** Multiply using the product rule: $(-9x^3y)(-2x^6y^4)$
1c. Divide using the quotient rule. $\dfrac{(-3)^6}{(-3)^3}$ $\dfrac{(-3)^6}{(-3)^3} = (-3)^{6-3} = (-3)^3$ or -27	**1c.** Divide using the quotient rule. $\dfrac{2^8}{2^4}$
1d. Divide using the quotient rule. $\dfrac{27x^{14}y^8}{3x^3y^5}$ $\dfrac{27x^{14}y^8}{3x^3y^5} = \dfrac{27}{3} \cdot x^{14-3} \cdot y^{8-5} = 9x^{11}y^3$	**1d.** Divide using the quotient rule. $\dfrac{25a^{13}b^4}{-5a^2b^3}$

1e. Evaluate -8^0.

Because there are no parentheses only 8 is raised to the 0 power.

$-8^0 = -(8^0) = -1$

1e. Evaluate $(-3)^0$.

1f. Write with a positive exponent. Simplify, if possible.

5^{-2}

$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

1f. Write with a positive exponent. Simplify, if possible.

4^{-3}

1g. Write with a positive exponent. Simplify, if possible.

$3x^{-6}y^4$

$3x^{-6}y^4 = 3 \cdot \frac{1}{x^6} \cdot y^4 = \frac{3y^4}{x^6}$

1g. Write with a positive exponent. Simplify, if possible.

$(4x^3)^{-2}$

1h. Simplify using the power rule.

$(3^3)^2$

$(3^3)^2 = 3^{3\cdot 2} = 3^6$ or 729

1h. Simplify using the power rule.

$(2^2)^3$

1i. Simplify using the power rule.

$(y^7)^{-2}$

$(y^7)^{-2} = y^{7(-2)} = y^{-14} = \frac{1}{y^{14}}$

1i. Simplify using the power rule.

$(x^{-5})^3$

1j. Simplify: $(-4x)^3$.

$(-4x)^3 = (-4)^3(x)^3 = -64x^3$

1j. Simplify: $(8x^3)^2$.

1k. Simplify: $\left(-\frac{2}{y}\right)^5$.

$\left(-\frac{2}{y}\right)^5 = \frac{(-2)^5}{y^5} = \frac{-32}{y^5} = -\frac{32}{y^5}$

1k. Simplify: $\left(-\frac{4}{x}\right)^3$.

Objective #2: Simplify exponential expressions.

✔ ***Solved Problem #2***

2. Simplify.

2a. $(2x^3y^6)^4$

$$\begin{aligned}(2x^3y^6)^4 &= (2)^4(x^3)^4(y^6)^4\\ &= 2^4x^{3\cdot4}y^{6\cdot4}\\ &= 16x^{12}y^{24}\end{aligned}$$

2b. $(-6x^2y^5)(3xy^3)$

$$\begin{aligned}(-6x^2y^5)(3xy^3) &= (-6)(3)x^2xy^5y^3\\ &= -18x^{2+1}y^{5+3}\\ &= -18x^3y^8\end{aligned}$$

Pencil Problem #2

2. Simplify.

2a. $(-3x^2y^5)^2$

2b. $(3x^4)(2x^7)$

2c. $\dfrac{100x^{12}y^2}{20x^{16}y^{-4}}$

$$\frac{100x^{12}y^2}{20x^{16}y^{-4}}=\left(\frac{100}{20}\right)\left(\frac{x^{12}}{x^{16}}\right)\left(\frac{y^2}{y^{-4}}\right)$$
$$=5x^{12-16}y^{2-(-4)}$$
$$=5x^{-4}y^6$$
$$=\frac{5y^6}{x^4}$$

2c. $\dfrac{24x^3y^5}{32x^7y^{-9}}$

2d. $\left(\dfrac{5x}{y^4}\right)^{-2}$

$$\left(\frac{5x}{y^4}\right)^{-2}=\frac{(5x)^{-2}}{(y^4)^{-2}}$$
$$=\frac{5^{-2}x^{-2}}{y^{-6}}$$
$$=\frac{y^6}{5^2x^2}$$
$$=\frac{y^6}{25x^2}$$

2d. $\left(\dfrac{5x^3}{y}\right)^{-2}$

Objective #3: Use scientific notation.

✔ *Solved Problem #3*

3a. Write in decimal notation:
-2.6×10^9

Move the decimal point 9 places to the right.
$-2.6\times10^9=-2,600,000,000$

✎ *Pencil Problem #3* ✎

3a. Write in decimal notation:
-7.16×10^6

3b. Write in decimal notation:
3.017×10^{-6}

Move the decimal point 6 places to the left.
$3.017\times10^{-6} = 0.000003017$

3b. Write in decimal notation:
7.9×10^{-1}

3c. Write in scientific notation: 5,210,000,000

The decimal point needs to be moved 9 places to the left.
$5,210,000,000 = 5.21\times10^{9}$

3c. Write in scientific notation: 32,000

3d. Write in scientific notation: –0.000000006893

The decimal point needs to be moved 8 places to the right.
$-0.00000006893 = -6.893\times10^{-8}$

3d. Write in scientific notation: –0.00000000504

3e. Perform the indicated computation. Write the answer in scientific notation.

$(7.1\times10^{5})(5\times10^{-7})$

$$\begin{aligned}(7.1\times10^{5})(5\times10^{-7}) &= (7.1\times5)\times10^{5+(-7)}\\ &= 35.5\times10^{-2}\\ &= (3.55\times10^{1})\times10^{-2}\\ &= 3.55\times10^{-1}\end{aligned}$$

3e. Perform the indicated computation. Write the answer in scientific notation.

$(1.6\times10^{15})(4\times10^{-11})$

3f. Perform the indicated computation. Write the answer in scientific notation. $\dfrac{1.2\times10^{6}}{3\times10^{-3}}$ $\dfrac{1.2\times10^{6}}{3\times10^{-3}}=\dfrac{1.2}{3}\times10^{6-(-3)}$ $=0.4\times10^{9}$ $=(4\times10^{-1})\times10^{9}$ $=4\times10^{8}$	**3f.** Perform the indicated computation. Write the answer in scientific notation. $\dfrac{2.4\times10^{-2}}{4.8\times10^{-6}}$

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1a. x^{10} *(P.2 #27)* **1b.** $18x^{9}y^{5}$ *(P.2 #47)* **1c.** 16 *(P.2 #17)* **1d.** $-5a^{11}b$ *(P.2 #51)*

1e. 1 *(P.2 #7)* **1f.** $\dfrac{1}{64}$ *(P.2 #11)* **1g.** $\dfrac{1}{16x^{6}}$ *(P.2 #55)*

1h. 64 *(P.2 #15)* **1i.** $\dfrac{1}{x^{15}}$ *(P.2 #33)* **1j.** $64x^{6}$ *(P.2 #39)* **1k.** $-\dfrac{64}{x^{3}}$ *(P.2 #41)*

2a. $9x^{4}y^{10}$ *(P.2 #43)* **2b.** $6x^{11}$ *(P.2 #45)* **2c.** $\dfrac{3y^{14}}{4x^{4}}$ *(P.2 #57)* **2d.** $\dfrac{y^{2}}{25x^{6}}$ *(P.2 #59)*

3a. −7,160,000 *(P.2 #69)* **3b.** 0.79 *(P.2 #71)* **3c.** 3.2×10^{4} *(P.2 #77)* **3d.** -5.04×10^{-9} *(P.2 #85)*
3e. 6.4×10^{4} *(P.2 #89)* **3f.** 5×10^{3} *(P.2 #101)*

Section P.3
Radicals and Rational Exponents

Radicals in Space?

What does space travel have to do with radicals?

Imagine that in the future we will be able to travel at velocities approaching the speed of light
(approximately 186,000 miles per second). According to Einstein's theory of special relativity, time would pass more quickly on Earth than it would in the moving spaceship.

Objective #1: Evaluate square roots.	
✔ ***Solved Problem #1***	✎ ***Pencil Problem #1*** ✎
1a. Evaluate $\sqrt{81}$. $\sqrt{81}=9$ Check: $9^2=81$	**1a.** Evaluate $\sqrt{36}$.
1b. Evaluate $-\sqrt{9}$. $-\sqrt{9}=-3$ Check: $(-3)^2=9$	**1b.** Evaluate $-\sqrt{36}$.
1c. Evaluate $\sqrt{\frac{1}{25}}$. $\sqrt{\frac{1}{25}}=\frac{1}{5}$ Check: $\left(\frac{1}{5}\right)^2=\frac{1}{25}$	**1c.** Evaluate $\sqrt{\frac{1}{81}}$.
1d. Evaluate $\sqrt{36+64}$. $\sqrt{36+64}=\sqrt{100}=10$	**1d.** Evaluate $\sqrt{25-16}$.

1e. Evaluate $\sqrt{36}+\sqrt{64}$.

$\sqrt{36}+\sqrt{64}=6+8=14$

1e. Evaluate $\sqrt{25}-\sqrt{16}$.

Objective #2: Simplify expressions of the form $\sqrt{a^2}$.

✔ *Solved Problem #2*

2. Evaluate $\sqrt{(-6)^2}$.

$\sqrt{(-6)^2}=|-6|=6$

✎ *Pencil Problem #2* ✎

2. Evaluate $\sqrt{(-13)^2}$.

Objective #3: Use the product rule to simplify square roots.

✔ *Solved Problem #3*

3a. Simplify $\sqrt{75}$.

$\sqrt{75}=\sqrt{25\cdot 3}=\sqrt{25}\cdot\sqrt{3}=5\sqrt{3}$

3b. Simplify $\sqrt{5x}\cdot\sqrt{10x}$.

$$\begin{aligned}\sqrt{5x}\cdot\sqrt{10x}&=\sqrt{5x\cdot 10x}\\&=\sqrt{50x^2}\\&=\sqrt{25x^2\cdot 2}\\&=\sqrt{25x^2}\sqrt{2}\\&=5x\sqrt{2}\end{aligned}$$

✎ *Pencil Problem #3* ✎

3a. Simplify $\sqrt{50}$.

3b. Simplify $\sqrt{2x}\cdot\sqrt{6x}$.

Objective #4: Use the quotient rule to simplify square roots.

✔ ***Solved Problem #4***

4a. Simplify $\sqrt{\frac{25}{16}}$.

$\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4}$

✎ ***Pencil Problem #4*** ✎

4a. Simplify $\sqrt{\frac{49}{16}}$.

✔ ***Solved Problem #4***

4b. Simplify $\frac{\sqrt{150x^3}}{\sqrt{2x}}$.

$$\frac{\sqrt{150x^3}}{\sqrt{2x}} = \sqrt{\frac{150x^3}{2x}}$$
$$= \sqrt{75x^2}$$
$$= \sqrt{25x^2 \cdot 3}$$
$$= \sqrt{25x^2}\sqrt{3}$$
$$= 5x\sqrt{3}$$

✎ ***Pencil Problem #4*** ✎

4b. Simplify $\frac{\sqrt{48x^3}}{\sqrt{3x}}$.

Objective #5: Add and subtract square roots.

✔ ***Solved Problem #5***

5a. Add: $8\sqrt{13} + 9\sqrt{13}$.

$8\sqrt{13} + 9\sqrt{13} = (8+9)\sqrt{13} = 17\sqrt{13}$

✎ ***Pencil Problem #5*** ✎

5a. Subtract: $6\sqrt{17x} - 8\sqrt{17x}$.

5b. Subtract: $6\sqrt{18x}-4\sqrt{8x}$.

$$\begin{aligned}6\sqrt{18x}-4\sqrt{8x}&=6\sqrt{9\cdot 2x}-4\sqrt{4\cdot 2x}\\&=6\cdot 3\sqrt{2x}-4\cdot 2\sqrt{2x}\\&=18\sqrt{2x}-8\sqrt{2x}\\&=(18-8)\sqrt{2x}\\&=10\sqrt{2x}\end{aligned}$$

5b. Add: $3\sqrt{18}+5\sqrt{50}$.

Objective #6: Rationalize denominators.

✔ *Solved Problem #6*

6a. Rationalize the denominator : $\dfrac{6}{\sqrt{12}}$.

Multiply by $\sqrt{3}$ to obtain the square root of a perfect square, $\sqrt{12}\cdot\sqrt{3}=\sqrt{36}$.

$$\frac{6}{\sqrt{12}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{6\sqrt{3}}{\sqrt{36}}=\frac{6\sqrt{3}}{6}=\sqrt{3}$$

6b. Rationalize the denominator : $\dfrac{8}{4+\sqrt{5}}$.

Multiply by $4-\sqrt{5}$, the conjugate of $4+\sqrt{5}$.

$$\begin{aligned}\frac{8}{4+\sqrt{5}}\cdot\frac{4-\sqrt{5}}{4-\sqrt{5}}&=\frac{8(4-\sqrt{5})}{4^2-(\sqrt{5})^2}\\&=\frac{8(4-\sqrt{5})}{16-5}\\&=\frac{8(4-\sqrt{5})}{11}\ \text{ or }\ \frac{32-8\sqrt{5}}{11}\end{aligned}$$

✎ *Pencil Problem #6* ✎

6a. Rationalize the denominator : $\dfrac{\sqrt{2}}{\sqrt{5}}$.

6b. Rationalize the denominator : $\dfrac{7}{\sqrt{5}-2}$.

Objective #7: Evaluate and perform operations with higher roots.

✔ Solved Problem #7

7a. Simplify $\sqrt[5]{8}\cdot\sqrt[5]{8}$.

$$\begin{aligned}\sqrt[5]{8}\cdot\sqrt[5]{8} &= \sqrt[5]{8\cdot 8}\\ &= \sqrt[5]{64}\\ &= \sqrt[5]{32\cdot 2}\\ &= \sqrt[5]{32}\cdot\sqrt[5]{2}\\ &= 2\sqrt[5]{2}\end{aligned}$$

7b. Simplify $\sqrt[3]{\dfrac{125}{27}}$.

$$\sqrt[3]{\frac{125}{27}} = \frac{\sqrt[3]{125}}{\sqrt[3]{27}} = \frac{5}{3}$$

7c. Subtract: $3\sqrt[3]{81}-4\sqrt[3]{3}$.

$$\begin{aligned}3\sqrt[3]{81}-4\sqrt[3]{3} &= 3\sqrt[3]{27\cdot 3}-4\sqrt[3]{3}\\ &= 3\cdot 3\sqrt[3]{3}-4\sqrt[3]{3}\\ &= 9\sqrt[3]{3}-4\sqrt[3]{3}\\ &= (9-4)\sqrt[3]{3}\\ &= 5\sqrt[3]{3}\end{aligned}$$

✎ Pencil Problem #7 ✎

7a. Simplify $\sqrt[3]{9}\cdot\sqrt[3]{6}$.

7b. Simplify $\dfrac{\sqrt[5]{64x^6}}{\sqrt[5]{2x}}$.

7c. Add: $5\sqrt[3]{16}+\sqrt[3]{54}$.

Objective #8: Understand and use rational exponents..	
✔ ***Solved Problem #8***	***Pencil Problem #8***
8a. Simplify $(-8)^{\frac{1}{3}}$. $(-8)^{\frac{1}{3}} = \sqrt[3]{-8} = -2$	**8a.** Simplify $36^{\frac{1}{2}}$.
8b. Simplify $32^{-\frac{2}{5}}$. $32^{-\frac{2}{5}} = \frac{1}{32^{\frac{2}{5}}} = \frac{1}{(\sqrt[5]{32})^2} = \frac{1}{2^2} = \frac{1}{4}$	**8b.** Simplify $125^{\frac{2}{3}}$.
8c. Simplify $\frac{20x^4}{5x^{\frac{3}{2}}}$. $\frac{20x^4}{5x^{\frac{3}{2}}} = \frac{20}{5} \cdot x^{4-\frac{3}{2}} = 4x^{\frac{8}{2}-\frac{3}{2}} = 4x^{\frac{5}{2}}$	**8c.** Simplify $(7x^{\frac{1}{3}})(2x^{\frac{1}{4}})$.
8d. Simplify $\sqrt[6]{x^3}$. $\sqrt[6]{x^3} = x^{\frac{3}{6}} = x^{\frac{1}{2}} = \sqrt{x}$	**8d.** Simplify $\sqrt[6]{x^4}$.

<u>**Answers** for Pencil Problems *(Textbook Exercise references in parentheses)*</u>:

1a. 6 *(P.3 #1)* **1b.** –6 *(P.3 #3)* **1c.** $\frac{1}{9}$ *(P.3 #23)* **1d.** 3 *(P.3 #7)* **1e.** 1 *(P.3 #9)*

2. 13 *(P.3 #11)* **3a.** $5\sqrt{2}$ *(P.3 #13)* **3b.** $2x\sqrt{3}$ *(P.3 #17)*

4a. $\frac{7}{4}$ *(P.3 #23)* **4b.** $4x$ *(P.3 #27)* **5a.** $-2\sqrt{17x}$ *(P.3 #35)* **5b.** $34\sqrt{2}$ *(P.3 #41)*

6a. $\frac{\sqrt{10}}{5}$ *(P.3 #47)* **6b.** $7(\sqrt{5}+2)$ *(P.3 #51)*

7a. $3\sqrt[3]{2}$ *(P.3 #71)* **7b.** $2x$ *(P.3 #73)* **7c.** $13\sqrt[3]{2}$ *(P.3 #77)*

8a. 6 *(P.3 #83)* **8b.** 25 *(P.3 #87)* **8c.** $14x^{\frac{7}{12}}$ *(P.3 #91)* **8d.** $\sqrt[3]{x^2}$ *(P.3 #105)*

Section P.4
Polynomials

What Are the Best Dimensions for a Box?

Many children get excited about gift boxes of all shapes and sizes, with the possible *exception* of clothing-sized boxes. (I must confess I dreaded boxes of that size.)

While completing the application exercises in this section of the textbook, we will use polynomials to model the dimensions of a box. We will then apply the concepts of this section to model the area of the box's base and its volume.

Objective #1: Understand the vocabulary of polynomials.

✔ *Solved Problem #1*

1. True or false: $7x^5 - 3x^3 + 8$ is a polynomial of degree 7 with three terms.

False. The expression $7x^5 - 3x^3 + 8$ is a polynomial with three terms, but its degree is 5, not 7.

✎ *Pencil Problem #1* ✎

1. True or false: $x^2 - 4x^3 + 9x - 12x^4 + 63$ is a polynomial of degree 2 with five terms.

Objective #2: Add and subtract polynomials.

✔ *Solved Problem #2*

2a. Add: $(-17x^3 + 4x^2 - 11x - 5) + (16x^3 - 3x^2 + 3x - 15)$.

$$(-17x^3 + 4x^2 - 11x - 5) + (16x^3 - 3x^2 + 3x - 15)$$
$$= (-17x^3 + 16x^3) + (4x^2 - 3x^2) + (-11x + 3x) + (-5 - 15)$$
$$= -x^3 + x^2 + (-8x) + (-20)$$
$$= -x^3 + x^2 - 8x - 20$$

✎ *Pencil Problem #2* ✎

2a. Add: $(-6x^3 + 5x^2 - 8x + 9) + (17x^3 + 2x^2 - 4x - 13)$.

2b. Subtract:
$(13x^3 - 9x^2 - 7x + 1) - (-7x^3 + 2x^2 - 5x + 9)$.

$(13x^3 - 9x^2 - 7x + 1) - (-7x^3 + 2x^2 - 5x + 9)$
$= (13x^3 - 9x^2 - 7x + 1) + (7x^3 - 2x^2 + 5x - 9)$
$= (13x^3 + 7x^3) + (-9x^2 - 2x^2) + (-7x + 5x) + (1 - 9)$
$= 20x^3 + (-11x^2) + (-2x) + (-8)$
$= 20x^3 - 11x^2 - 2x - 8$

2b. Subtract:
$(17x^3 - 5x^2 + 4x - 3) - (5x^3 - 9x^2 - 8x + 11)$.

Objective #3: Multiply polynomials.

✔ *Solved Problem #3*

3. Multiply: $(5x - 2)(3x^2 - 5x + 4)$.

$(5x - 2)(3x^2 - 5x + 4)$
$= 5x(3x^2 - 5x + 4) - 2(3x^2 - 5x + 4)$
$= 5x \cdot 3x^2 + 5x(-5x) + 5x \cdot 4 - 2 \cdot 3x^2 - 2(-5x) - 2 \cdot 4$
$= 15x^3 - 25x^2 + 20x - 6x^2 + 10x - 8$
$= 15x^3 - 31x^2 + 30x - 8$

✎ *Pencil Problem #3* ✐

3. Multiply: $(2x - 3)(x^2 - 3x + 5)$.

Objective #4: Use FOIL in polynomial multiplication.

✔ *Solved Problem #4*

4. Multiply: $(7x - 5)(4x - 3)$.
Use FOIL.

First: $7x \cdot 4x$ Outside: $7x(-3)$
Inside: $-5 \cdot 4x$ Last: $-5(-3)$
$(7x - 5)(4x - 3)$
$= 7x \cdot 4x + 7x(-3) - 5 \cdot 4x - 5(-3)$
$= 28x^2 - 21x - 20x + 15$
$= 28x^2 - 41x + 15$

✎ *Pencil Problem #4* ✐

4. Multiply: $(3x + 5)(2x + 1)$.

Objective #5: Use special products in polynomial multiplication.

✔ *Solved Problem #5*

5a. Multiply: $(7x+8)(7x-8)$.

Use $(A+B)(A-B)=A^2-B^2$.

$$(7x+8)(7x-8)=(7x)^2-8^2$$
$$=49x^2-64$$

5b. Multiply: $(5x+4)^2$.

Use $(A+B)^2=A^2+2AB+B^2$.

$$(5x+4)^2=(5x)^2+2(5x)(4)+4^2$$
$$=25x^2+40x+16$$

5c. Multiply: $(x-9)^2$.

Use $(A-B)^2=A^2-2AB+B^2$.

$$(x-9)^2=x^2-2\cdot x\cdot 9+9^2$$
$$=x^2-18x+81$$

✎ *Pencil Problem #5* ✎

5a. Multiply: $(5-7x)(5+7x)$.

5b. Multiply: $(2x+3)^2$.

5c. Multiply: $(x-3)^2$.

Objective #6: Perform operations with polynomials in several variables.

✔ *Solved Problem #6*

6a. Multiply: $(7x-6y)(3x-y)$.

Use FOIL.

$$(7x-6y)(3x-y)$$
$$=7x\cdot 3x+7x(-y)-6y\cdot 3x-6y(-y)$$
$$=21x^2-7xy-18xy+6y^2$$
$$=21x^2-25xy+6y^2$$

✎ *Pencil Problem #6* ✎

6a. Multiply: $(x+5y)(7x+3y)$.

6b. Multiply: $(2x+4y)^2$. Use $(A+B)^2 = A^2+2AB+B^2$. $(2x+4y)^2 = (2x)^2+2(2x)(4y)+(4y)^2$ $= 4x^2+16xy+16y^2$	**6b.** Multiply: $(7x+5y)^2$.
6c. Multiply: $(3x+2+5y)(3x+2-5y)$. Group terms and use $(A+B)(A-B)=A^2-B^2$. $(3x+2+5y)(3x+2-5y)$ $=[(3x+2)+5y][(3x+2)-5y]$ $=(3x+2)^2-(5y)^2$ $=(3x)^2+2(3x)(2)+2^2-25y^2$ $=9x^2+12x+4-25y^2$	**6c.** Multiply: $(x+y+3)(x+y-3)$.
6d. Multiply: $(2x+y+3)^2$. Group terms and use $(A+B)^2 = A^2+2AB+B^2$. $(2x+y+3)^2 = [(2x+y)+3]^2$ $(2x+y)^2+2(2x+y)(3)+3^2$ $=(2x)^2+2(2x)(y)+y^2+12x+6y+9$ $=4x^2+4xy+y^2+12x+6y+9$	**6d.** Multiply: $(x+y+1)^2$.

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1. False *(P.4 #7)* **2a.** $11x^3+7x^2-12x-4$ *(P.4 #9)* **2b.** $12x^3+4x^2+12x-14$ *(P.4 #11)*

3. $2x^3-9x^2+19x-15$ *(P.4 #17)* **4.** $6x^2+13x+5$ *(P.4 #23)*

5a. $25-49x^2$ *(P.4 #35)* **5b.** $4x^2+12x+9$ *(P.4 #43)* **5c.** x^2-6x+9 *(P.4 #45)*

6a. $7x^2+38xy+15y^2$ *(P.4 #59)* **6b.** $49x^2+70xy+25y^2$ *(P.4 #65)*

6c. $x^2+2xy+y^2-9$ *(P.4 #73)* **6d.** $x^2+2xy+y^2+2x+2y+1$ *(P.4 #79)*

Section P.5
Factoring Polynomials

What's the sales price?

Many times retailers advertise their discounts in terms of percentages by which the price is reduced, such as 30% off. If a product still doesn't sell, the retailer may offer an additional 30% off the price that has already been reduced by 30%.

In this section's Exercise Set, you will see how the 30% discount followed by another 30% discount can be expressed as a polynomial. By factoring the polynomial and simplifying, you will see that our double discount means that we pay 49% of the original price.

Objective #1: Factor out the greatest common factor.

✔ *Solved Problem #1*

1a. Factor $10x^3 - 4x^2$.

2 is the greatest integer that divides 10 and 4. x^2 is the greatest expression that divides x^3 and x^2. The GCF is $2x^2$.

$$10x^3 - 4x^2 = 2x^2(5x) - 2x^2(2)$$
$$= 2x^2(5x - 2)$$

1b. Factor $2x(x-7)+3(x-7)$.

The GCF is the binomial factor $(x - 7)$.

$$2x(x-7)+3(x-7) = (x-7)(2x+3)$$

✎ *Pencil Problem #1* ✐

1a. Factor $3x^2 + 6x$.

1b. Factor $x(x+5)+3(x+5)$.

Objective #2: Factor by grouping.

✔ *Solved Problem #2*

2. Factor $x^3 + 5x^2 - 2x - 10$.

The GCF of the first two terms is x^2, and the GCF of the last two terms is −2. After factoring out these GCFs, factor out the common binomial factor.

$$\begin{aligned} x^3 + 5x^2 - 2x - 10 &= (x^3 + 5x^2) + (-2x - 10) \\ &= x^2(x+5) - 2(x+5) \\ &= (x+5)(x^2 - 2) \end{aligned}$$

✎ *Pencil Problem #2* ✎

2. Factor $x^3 - 2x^2 + 5x - 10$.

Objective #3: Factor trinomials.

✔ *Solved Problem #3*

3a. Factor $x^2 - 5x - 14$.

The leading coefficient is 1. We look for factors of −14 that sum to −5.

−7(2) = −14 and −7 + 2 = −5
The numbers are −7 and 2.
$x^2 - 5x - 14 = (x-7)(x+2)$

✎ *Pencil Problem #3* ✎

3a. Factor $x^2 - 8x + 15$.

3b. Factor $6x^2 + 19x - 7$.

The leading coefficient is 6, not 1. $6x^2$ factors as $6x(x)$ or $3x(2x)$. −7 factors as −7(1) or 7(−1).

The possible factorizations are

$(6x - 7)(x + 1)$	$(6x + 1)(x - 7)$
$(6x + 7)(x - 1)$	$(6x - 1)(x + 7)$
$(3x - 7)(2x + 1)$	$(3x + 1)(2x - 7)$
$(3x + 7)(2x - 1)$	$(3x - 1)(2x + 7)$

We want the combination, if there is one, that results in a sum of Outside and Inside terms of $19x$. Compute the sums of the Outside and Inside terms in the possible factorizations until you find one that results in $19x$.

For $(3x - 1)(2x + 7)$:
Outside: $3x(7) = 21x$
Inside: $-1(2x) = -2x$
Sum: $21x + (-2x) = 19x$
So, $6x^2 + 19x - 7 = (3x - 1)(2x + 7)$.

3b. Factor $9x^2 - 9x + 2$.

Objective #4: Factor the difference of squares.

✔ *Solved Problem #4*

4. Factor: $36x^2 - 25$.

Note that $36x^2 = (6x)^2$ and $25 = 5^2$ can both be expressed as squares.

Use $A^2 - B^2 = (A + B)(A - B)$.

$$\begin{aligned} 36x^2 - 25 &= (6x)^2 - 5^2 \\ &= (6x + 5)(6x - 5) \end{aligned}$$

✎ *Pencil Problem #4* ✐

4. Factor $9x^2 - 25y^2$.

Objective #5: Factor perfect square trinomials.

✔ ***Solved Problem #5***

5a. Factor $x^2 + 14x + 49$.

Note that the first term is the square of x, the last term is the square of 7, and the middle term is twice the product of x and 7.

Factor using $A^2 + 2AB + B^2 = (A + B)^2$.

$$x^2 + 14x + 49 = x^2 + 2 \cdot x \cdot 7 + 7^2$$
$$= (x + 7)^2$$

5b. Factor $16x^2 - 56x + 49$.

Note that the first term is the square of $4x$, the last term is the square of 7, and the middle term is twice the product of $4x$ and 7.
Factor using $A^2 - 2AB + B^2 = (A - B)^2$.

$$16x^2 - 56x + 49 = (4x)^2 - 2 \cdot 4x \cdot 7 + 7^2$$
$$= (4x - 7)^2$$

✎ ***Pencil Problem #5*** ✐

5a. Factor $x^2 + 2x + 1$.

5b. Factor $9x^2 - 6x + 1$.

Objective #6: Factor the sum or difference of two cubes.

✔ ***Solved Problem #6***

6a. Factor $x^3 + 1$.

Note that both terms can be expressed as cubes.
Factor using $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$.

$$x^3 + 1 = x^3 + 1^3$$
$$= (x + 1)(x^2 - x \cdot 1 + 1^2)$$
$$= (x + 1)(x^2 - x + 1)$$

✎ ***Pencil Problem #6*** ✐

6a. Factor $x^3 + 27$.

6b. Factor $125x^3 - 8$.

Note that the first term is the cube of $5x$ and the second term is the cube of 2.

Factor using $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$.

$$125x^3 - 8 = (5x)^3 - 2^3$$
$$= (5x - 2)[(5x)^2 + 5x \cdot 2 + 2^2]$$
$$= (5x - 2)(25x^2 + 10x + 4)$$

6b. Factor $8x^3 - 1$.

Objective #7: Use a general strategy for factoring polynomials..

✔ *Solved Problem #7*

7a. Factor $3x^3 - 30x^2 + 75x$.

First, factor out the GCF, $3x$.

$$3x^3 - 30x^2 + 75x = 3x(x^2 - 10x + 25)$$

Now factor the trinomial. Find factors of 25 that sum to –10, or use the formula for a perfect square trinomial, $A^2 - 2AB + B^2 = (A - B)^2$.

$$3x^3 - 30x^2 + 75x = 3x(x^2 - 10x + 25)$$
$$= 3x(x^2 - 2 \cdot x \cdot 5 + 5^2)$$
$$= 3x(x - 5)^2$$

7b. Factor $x^2 - 36a^2 + 20x + 100$.

Regroup factors and look for opportunities to factor within groupings.

$$x^2 - 36a^2 + 20x + 100 = (x^2 + 20x + 100) - 36a^2$$

Factor the expression in parentheses using $A^2 + 2AB + B^2 = (A + B)^2$.

$$(x^2 + 20x + 100) - 36a^2 = (x^2 + 2 \cdot x \cdot 10 + 10^2) - 36a^2$$
$$= (x + 10)^2 - 36a^2$$

✎ *Pencil Problem #7* ✎

7a. Factor $20y^4 - 45y^2$.

7b. Factor $x^2 - 12x + 36 - 49y^2$.

This last form is the difference of squares. Factor using $A^2 - B^2 = (A+B)(A-B)$.

$$\begin{aligned}(x+10)^2 - 36a^2 &= (x+10)^2 - (6a)^2 \\ &= [(x+10)+6a][(x+10)-6a] \\ &= (x+10+6a)(x+10-6a)\end{aligned}$$

So, $x^2 - 36a^2 + 20x + 100 = (x+10+6a)(x+10-6a)$.

Objective #8: Factor algebraic expressions containing fractional and negative exponents.

✔ *Solved Problem #8*

8. Factor and simplify: $x(x-1)^{-\frac{1}{2}} + (x-1)^{\frac{1}{2}}$.

The GCF is $(x - 1)$ with the smaller exponent. Thus, the GCF is $(x-1)^{-\frac{1}{2}}$.

$$\begin{aligned}x(x-1)^{-\frac{1}{2}} + (x-1)^{\frac{1}{2}} &= (x-1)^{-\frac{1}{2}} \cdot x + (x-1)^{-\frac{1}{2}} \cdot (x-1) \\ &= (x-1)^{-\frac{1}{2}}[x+(x-1)] \\ &= (x-1)^{-\frac{1}{2}}(2x-1) \\ &= \frac{2x-1}{(x-1)^{1/2}}\end{aligned}$$

✎ *Pencil Problem #8* ✐

8. Factor and simplify: $(x+3)^{\frac{1}{2}} - (x+3)^{\frac{3}{2}}$.

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1a. $3x(x+2)$ *(P.5 #3)* **1b.** $(x+5)(x+3)$ *(P.5 #7)* **2.** $(x-2)(x^2+5)$ *(P.5 #11)*

3a. $(x-5)(x-3)$ *(P.5 #21)* **3b.** $(3x-2)(3x-1)$ *(P.5 #31)*

4. $(3x-5y)(3x+5y)$ *(P.5 #43)* **5a.** $(x+1)^2$ *(P.5 #49)* **5b.** $(3x-1)^2$ *(P.5 #55)*

6a. $(x+3)(x^2-3x+9)$ *(P.5 #57)* **6b.** $(2x-1)(4x^2+2x+1)$ *(P.5 #61)*

7a. $5y^2(2y+3)(2y-3)$ *(P.5 #83)* **7b.** $(x-6+7y)(x-6-7y)$ *(P.5 #85)* **8.** $-(x+3)^{\frac{1}{2}}(x+2)$ *(P.5 #97)*

Section P.6
Rational Expressions

Ouch! That Hurts!!

Though it may not be fun to get a flu shot, it is a great way to protect yourself from getting sick!

In this section of the textbook, one of the application problems will explore the costs for inoculating various percentages of the population.

Objective #1: Specify numbers that must be excluded from the domain of a rational expression.

✔ *Solved Problem #1*

1. Find all real numbers that must be excluded from the domain of each rational expression.

$$\frac{7x}{x^2-5x-14}$$

Factor the denominator.

$$x^2-5x-14=(x-7)(x+2)$$

The first factor would be 0 if $x = 7$. The second factor would be 0 if $x = -2$. We must exclude -2 and 7 from the domain.

✎ *Pencil Problem #1* ✎

1. Find all real numbers that must be excluded from the domain of each rational expression.

$$\frac{x+5}{x^2-25}$$

Objective #2: Simplify rational expressions.

✔ *Solved Problem #2*

2a. Simplify $\frac{x^3+3x^2}{x+3}$.

Note that $x \neq -3$ since -3 would make the denominator 0.

Factor the numerator and divide out common factors.

$$\frac{x^3+3x^2}{x+3}=\frac{x^2(x+3)}{x+3}=\frac{x^2\,\cancel{(x+3)}}{\cancel{x+3}}$$

$$=x^2,\ x \neq -3$$

✎ *Pencil Problem #2* ✎

2a. Simplify $\frac{3x-9}{x^2-6x+9}$.

2b. Simplify $\dfrac{x^2-1}{x^2+2x+1}$.

By factoring the denominator, $x^2+2x+1=(x+1)^2$, we see that $x \neq -1$.

Factor the numerator and denominator and divide out common factors.

$$\frac{x^2-1}{x^2+2x+1}=\frac{(x+1)(x-1)}{(x+1)(x+1)}=\frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}(x+1)}$$
$$=\frac{x-1}{x+1},\ x\neq -1$$

2b. Simplify $\dfrac{y^2+7y-18}{y^2-3y+2}$.

Objective #3: Multiply rational expressions.

✔ *Solved Problem #3*

3. Multiply: $\dfrac{x+3}{x^2-4}\cdot\dfrac{x^2-x-6}{x^2+6x+9}$.

Factor and divide by common factors.

$$\frac{x+3}{x^2-4}\cdot\frac{x^2-x-6}{x^2+6x+9}=\frac{x+3}{(x+2)(x-2)}\cdot\frac{(x-3)(x+2)}{(x+3)(x+3)}$$
$$=\frac{\cancel{x+3}}{\cancel{(x+2)}(x-2)}\cdot\frac{(x-3)\cancel{(x+2)}}{\cancel{(x+3)}(x+3)}$$
$$=\frac{x-3}{(x-2)(x+3)},\ x\neq -3,-2,2$$

To see which values must be excluded from the domain, look at the factored forms of the denominators in the second step.

✎ *Pencil Problem #3* ✎

3. Multiply: $\dfrac{x^2-5x+6}{x^2-2x-3}\cdot\dfrac{x^2-1}{x^2-4}$.

Objective #4: Divide rational expressions.

✔ *Solved Problem #4*

4. Divide: $\dfrac{x^2-2x+1}{x^3+x}\div\dfrac{x^2+x-2}{3x^2+3}$.

Invert the divisor and multiply.

$$\frac{x^2-2x+1}{x^3+x}\div\frac{x^2+x-2}{3x^2+3}=\frac{x^2-2x+1}{x^3+x}\cdot\frac{3x^2+3}{x^2+x-2}$$
$$=\frac{(x-1)\cancel{(x-1)}}{x\cancel{(x^2+1)}}\cdot\frac{3\cancel{(x^2+1)}}{(x+2)\cancel{(x-1)}}$$
$$=\frac{3(x-1)}{x(x+2)},\ x\neq -2,0,1$$

✎ *Pencil Problem #4* ✎

4. Divide: $\dfrac{x^2-25}{2x-2}\div\dfrac{x^2+10x+25}{x^2+4x-5}$.

Objective #5: Add and subtract rational expressions.

✔ Solved Problem #5

5a. Subtract: $\frac{x}{x+1}-\frac{3x+2}{x+1}$.

The expressions have the same denominator. Subtract numerators.

$$\frac{x}{x+1}-\frac{3x+2}{x+1}=\frac{x-(3x+2)}{x+1}$$
$$=\frac{x-3x-2}{x+1}$$
$$=\frac{-2x-2}{x+1}$$
$$=\frac{-2\cancel{(x+1)}}{\cancel{x+1}}$$
$$=-2,\ x\neq -1$$

Pencil Problem #5

5a. Add: $\frac{4x+1}{6x+5}+\frac{8x+9}{6x+5}$.

5b. Subtract: $\frac{x}{x^2-10x+25}-\frac{x-4}{2x-10}$.

Factor the denominators.

$x^2-10x+25=(x-5)(x-5)$

$2x-10=2(x-5)$

LCD = $2(x-5)(x-5)$

$$\frac{x}{x^2-10x+25}-\frac{x-4}{2x-10}$$
$$=\frac{x}{(x-5)(x-5)}-\frac{x-4}{2(x-5)}$$
$$=\frac{2x}{2(x-5)(x-5)}-\frac{(x-4)(x-5)}{2(x-5)(x-5)}$$
$$=\frac{2x-(x-4)(x-5)}{2(x-5)^2}$$
$$=\frac{2x-(x^2-9x+20)}{2(x-5)^2}$$
$$=\frac{2x-x^2+9x-20}{2(x-5)^2}$$
$$=\frac{-x^2+11x-20}{2(x-5)^2},\ x\neq 5$$

5b. Subtract: $\frac{3x}{x^2+3x-10}-\frac{2x}{x^2+x-6}$.

5c. Add: $\frac{3}{x+1}+\frac{5}{x-1}$.

The denominators are not equal and have no common factor. Use the property $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$.

$$\begin{aligned}\frac{3}{x+1}+\frac{5}{x-1}&=\frac{3(x-1)+5(x+1)}{(x+1)(x-1)}\\&=\frac{3x-3+5x+5}{(x+1)(x-1)}\\&=\frac{8x+2}{(x+1)(x-1)}\\&=\frac{2(4x+1)}{(x+1)(x-1)},\ x\neq -1,1\end{aligned}$$

5c. Add: $\frac{3}{x+4}+\frac{6}{x+5}$.

Objective #6: Simplify complex rational expressions.

✔ *Solved Problem #6*

6a. Simplify: $\dfrac{\frac{1}{x}-\frac{3}{2}}{\frac{1}{x}+\frac{3}{4}}$.

Subtract and add in the numerator and denominator to obtain a single rational expression in each.

$$\frac{1}{x}-\frac{3}{2}=\frac{1\cdot 2}{x\cdot 2}-\frac{3\cdot x}{2\cdot x}=\frac{2}{2x}-\frac{3x}{2x}=\frac{2-3x}{2x}$$
$$\frac{1}{x}+\frac{3}{4}=\frac{1\cdot 4}{x\cdot 4}+\frac{3\cdot x}{4\cdot x}=\frac{4}{4x}+\frac{3x}{4x}=\frac{4+3x}{4x}$$

Now return to the original complex fraction.

$$\begin{aligned}\frac{\frac{1}{x}-\frac{3}{2}}{\frac{1}{x}+\frac{3}{4}}&=\frac{\frac{2-3x}{2x}}{\frac{4+3x}{4x}}\\&=\frac{2-3x}{2x}\cdot\frac{4x}{4+3x}\\&=\frac{2-3x}{\cancel{2x}}\cdot\frac{2\cdot\cancel{2x}}{4+3x}\\&=\frac{2(2-3x)}{4+3x},\ x\neq 0,-\frac{4}{3}\end{aligned}$$

✎ *Pencil Problem #6* ✎

6a. Simplify: $\dfrac{1+\frac{1}{x}}{3-\frac{1}{x}}$.

6b. Simplify: $\dfrac{\dfrac{1}{x+7}-\dfrac{1}{x}}{7}$.

The LCD of the fractions within the complex fraction is $x(x + 7)$. Multiply the numerator and the denominator of the complex fraction by the LCD.

$$\frac{\frac{1}{x+7}-\frac{1}{x}}{7}=\frac{\left(\frac{1}{x+7}-\frac{1}{x}\right)x(x+7)}{7x(x+7)}$$

$$=\frac{\frac{1}{x+7}\cdot x(x+7)-\frac{1}{x}\cdot x(x+7)}{7x(x+7)}$$

$$=\frac{x-(x+7)}{7x(x+7)}$$

$$=\frac{-7}{7x(x+7)}$$

$$=\frac{-1}{x(x+7)},\ x\neq -7,0$$

6b. Simplify: $\dfrac{\dfrac{3}{x-2}-\dfrac{4}{x+2}}{\dfrac{7}{x^2-4}}$.

Objective #7: Simplify rational expressions that occur in calculus.

✔ *Solved Problem #7*

7. Simplify: $\dfrac{\sqrt{x}+\dfrac{1}{\sqrt{x}}}{x}$.

Multiply the numerator and denominator by $\sqrt{x}$.

$$\frac{\sqrt{x}+\frac{1}{\sqrt{x}}}{x}=\frac{\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)}{x}\cdot\frac{\sqrt{x}}{\sqrt{x}}$$

$$=\frac{\sqrt{x}\sqrt{x}+\frac{1}{\sqrt{x}}\sqrt{x}}{x\sqrt{x}}$$

$$=\frac{x+1}{\sqrt{x^3}}$$

Note that the simplification in the denominator above can be done as follows:

$$x\sqrt{x}=x\cdot x^{\frac{1}{2}}=x^{1+\frac{1}{2}}=x^{\frac{3}{2}}=\sqrt{x^3}.$$

✎ *Pencil Problem #7* ✎

7. Simplify: $\dfrac{\sqrt{x}-\dfrac{1}{3\sqrt{x}}}{\sqrt{x}}$.

Objective #8: Rationalize numerators.

✔ *Solved Problem #8*

8. Rationalize the numerator: $\dfrac{\sqrt{x+3}-\sqrt{x}}{3}$.

Multiply the numerator and the denominator by the conjugate of the numerator.

$$\frac{\sqrt{x+3}-\sqrt{x}}{3}=\frac{\sqrt{x+3}-\sqrt{x}}{3}\cdot\frac{\sqrt{x+3}+\sqrt{x}}{\sqrt{x+3}+\sqrt{x}}$$
$$=\frac{(\sqrt{x+3})^2-(\sqrt{x})^2}{3(\sqrt{x+3}+\sqrt{x})}$$
$$=\frac{x+3-x}{3(\sqrt{x+3}+\sqrt{x})}$$
$$=\frac{3}{3(\sqrt{x+3}+\sqrt{x})}$$
$$=\frac{1}{\sqrt{x+3}+\sqrt{x}}$$

✎ *Pencil Problem #8* ✎

8. Rationalize the numerator: $\dfrac{\sqrt{x+5}-\sqrt{x}}{5}$.

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1. $-5, 5$ *(P.6 #3)* **2a.** $\dfrac{3}{x-3},\ x\neq 3$ *(P.6 #7)* **2b.** $\dfrac{y+9}{y-1},\ y\neq 1,2$ *(P.6 #11)*

3. $\dfrac{x-1}{x+2},\ x\neq -2,-1,2,3$ *(P.6 #19)* **4.** $\dfrac{x-5}{2},\ x\neq -5,1$ *(P.6 #29)*

5a. $2,\ x\neq -\dfrac{5}{6}$ *(P.6 #33)* **5b.** $\dfrac{x^2-x}{(x+5)(x-2)(x+3)},\ x\neq -5,-3,2$ *(P.6 #53)*

5c. $\dfrac{9x+39}{(x+4)(x+5)},\ x\neq -5,-4$ *(P.6 #41)*

6a. $\dfrac{x+1}{3x-1},\ x\neq 0,\dfrac{1}{3}$ *(P.6 #61)* **6b.** $-\dfrac{x-14}{7},\ x\neq -2,2$ *(P.6 #67)*

7. $1-\dfrac{1}{3x}, x>0$ *(P.6 #73)* **8.** $\dfrac{1}{\sqrt{x+5}+\sqrt{x}}$ *(P.6 #79)*

Section P.7
Equations

Maybe I Should Ride the Bus Instead

Did you know that the likelihood that a driver will be involved in a fatal crash decreases with age until about age 45 and then increases after that? Formulas that model data that first decrease and then increase contain a variable squared. When we use these models to answer questions about the data, we often need to find the solutions of a *quadratic equation*.

Quadratic equations may have exactly two distinct solutions. Thus, when we find the age at which drivers are involved in 3 fatal crashes per 100 million miles driven, we will find two different ages, one less 45 and the other greater than 45.

Objective #1: Solve linear equations in one variable

✔ *Solved Problem #1*

1. Solve and check: $4(2x+1)=29+3(2x-5)$

Simplify the algebraic expression on each side.

$$4(2x+1)=29+3(2x-5)$$
$$8x+4=29+6x-15$$
$$8x+4=6x+14$$

Collect variable terms on one side and constant terms on the other side.

$$8x-6x+4=6x-6x+14$$
$$2x+4=14$$
$$2x+4-4=14-4$$

Isolate the variable and solve.

$$\frac{2x}{2}=\frac{10}{2}$$
$$x=5$$

Check:

$$4(2x+1)=29+3(2x-5)$$
$$4(2\cdot5+1)=29+3(2\cdot5-5)$$
$$4(11)=29+3(5)$$
$$44=44$$

The solution set is $\{5\}$.

✎ *Pencil Problem #1* ✐

1. Solve and check: $3(x-2)+7=2(x+5)$

Objective #2: Solve linear equations containing fractions.

✔ *Solved Problem #2*

2. Solve and check: $\frac{x-3}{4}=\frac{5}{14}-\frac{x+5}{7}$

The LCD is 28.

$$\frac{x-3}{4}=\frac{5}{14}-\frac{x+5}{7}$$
$$28\left(\frac{x-3}{4}\right)=28\left(\frac{5}{14}-\frac{x+5}{7}\right)$$
$$\frac{28}{1}\left(\frac{x-3}{4}\right)=\frac{28}{1}\left(\frac{5}{14}\right)-\frac{28}{1}\left(\frac{x+5}{7}\right)$$
$$7(x-3)=2(5)-4(x+5)$$
$$7x-21=10-4x-20$$
$$7x-21=-4x-10$$
$$7x+4x-21=-4x+4x-10$$
$$11x-21=-10$$
$$11x-21+21=-10+21$$
$$11x=11$$
$$\frac{11x}{11}=\frac{11}{11}$$
$$x=1$$

Check:

$$\frac{x-3}{4}=\frac{5}{14}-\frac{x+5}{7}$$
$$\frac{1-3}{4}=\frac{5}{14}-\frac{1+5}{7}$$
$$\frac{-2}{4}=\frac{5}{14}-\frac{6}{7}$$
$$-\frac{1}{2}=-\frac{1}{2}$$

The solution set is $\{1\}$.

✎ *Pencil Problem #2* ✎

2. Solve and check: $\frac{x+3}{6}=\frac{3}{8}+\frac{x-5}{4}$

Objective #3: Solve rational equations with variables in denominators.

✔ *Solved Problem #3a*

3a. Solve: $\frac{6}{x+3}-\frac{5}{x-2}=\frac{-20}{x^2+x-6}$

By factoring $x^2+x-6=(x+3)(x-2)$, we see that the LCD is $(x+3)(x-2)$ and that $x\neq -3$ and $x\neq 2$.

$$\frac{6}{x+3}-\frac{5}{x-2}=\frac{-20}{x^2+x-6},\ x\neq -3,\ x\neq 2$$

$$(x+3)(x-2)\left(\frac{6}{x+3}-\frac{5}{x-2}\right)=(x+3)(x-2)\cdot\frac{-20}{(x+3)(x-2)}$$

$$\cancel{(x+3)}(x-2)\cdot\frac{6}{\cancel{x+3}}-(x+3)\cancel{(x-2)}\cdot\frac{5}{\cancel{x-2}}=\cancel{(x+3)}\cancel{(x-2)}\cdot\frac{-20}{\cancel{(x+3)}\cancel{(x-2)}}$$

$$6(x-2)-5(x+3)=-20$$
$$6x-12-5x-15=-20$$
$$x-27=-20$$
$$x-27+27=-20+27$$
$$x=7$$

You can check the proposed solution x = 7 in the original equation. Since $x = 7$ is not part of the restriction $x\neq -3$ and $x\neq 2$, the solution set is $\{7\}$.

✎ *Pencil Problem #3a* ✐

3a. Solve: $\frac{2}{x+1}-\frac{1}{x-1}=\frac{2x}{x^2-1}$

✔ *Solved Problem #3b*

3b. Solve: $\frac{1}{x+2}=\frac{4}{x^2-4}-\frac{1}{x-2}$

By factoring $x^2-4=(x+2)(x-2)$, we see that the LCD is $(x+2)(x-2)$ and that $x \neq -2$ and $x \neq 2$.

$$\frac{1}{x+2}=\frac{4}{x^2-4}-\frac{1}{x-2},\ x\neq -2,\ x\neq 2$$

$$(x+2)(x-2)\cdot\frac{1}{x+2}=(x+2)(x-2)\left(\frac{4}{(x+2)(x-2)}-\frac{1}{x-2}\right)$$

$$\cancel{(x+2)}(x-2)\cdot\frac{1}{\cancel{x+2}}=\cancel{(x+2)}\,\cancel{(x-2)}\cdot\frac{4}{\cancel{(x+2)}\,\cancel{(x-2)}}-(x+2)\cancel{(x-2)}\cdot\frac{1}{\cancel{x-2}}$$

$$1(x-2)=4-1(x+2)$$
$$x-2=4-x-2$$
$$x-2=-x+2$$
$$x-2+x=-x+2+x$$
$$2x-2=2$$
$$2x-2+2=2+2$$
$$2x=4$$
$$\frac{2x}{2}=\frac{4}{2}$$
$$x=2$$

Since $x = 2$ is part of the restriction $x \neq -2$ and $x \neq 2$, the solution set is $\varnothing$, the empty set.

✎ *Pencil Problem #3b* ✐

3b. Solve: $\frac{1}{x-4}-\frac{5}{x+2}=\frac{6}{x^2-2x-8}$

Objective #4: Solve a formula for a variable.

✔ *Solved Problem #4*

4. Solve for q: $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$

The LCD is pqf.

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

$$pqf\left(\frac{1}{p}+\frac{1}{q}\right)=pqf\cdot\frac{1}{f}$$

$$\cancel{p}qf\cdot\frac{1}{\cancel{p}}+p\cancel{q}f\cdot\frac{1}{\cancel{q}}=pq\cancel{f}\cdot\frac{1}{\cancel{f}}$$

$$qf+pf=pq$$

$$qf+pf-qf=pq-qf$$

$$pf=q(p-f)$$

$$\frac{pf}{p-f}=\frac{q(p-f)}{p-f}$$

$$\frac{pf}{p-f}=q \text{ or } q=\frac{pf}{p-f}$$

Pencil Problem #4

4. Solve for f: $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$

Objective #5: Solve equations involving absolute value.

✔ *Solved Problem #5*

5. Solve: $4|1-2x|-20=0$

First isolate the absolute value expression.

$$4|1-2x|-20=0$$

$$4|1-2x|=20$$

$$|1-2x|=5$$

Now rewrite $|u|=c$ as $u=c$ or $u=-c$.

$1-2x=5$ or $1-2x=-5$

$-2x=4$ $\quad$ $-2x=-6$

$x=-2$ $\quad$ $x=3$

You can check −2 and 3 in the original equation. The solution set is $\{-2, 3\}$.

Pencil Problem #5

5. Solve: $2|3x-2|=14$

Objective #6: Solve quadratic equations by factoring.

✔ *Solved Problem #6*

6. Solve by factoring.

6a. $3x^2 - 9x = 0$

$$3x^2 - 9x = 0$$
$$3x(x - 3) = 0$$
$$3x = 0 \text{ or } x - 3 = 0$$
$$x = 0 \text{ or } \quad x = 3$$

The solution set is $\{0, 3\}$.

6b. $2x^2 + x = 1$

$$2x^2 + x = 1$$
$$2x^2 + x - 1 = 0$$
$$(2x - 1)(x + 1) = 0$$
$$2x - 1 = 0 \text{ or } x + 1 = 0$$
$$2x = 1 \text{ or } \quad x = -1$$
$$x = \frac{1}{2}$$

The solution set is $\left\{-1, \frac{1}{2}\right\}$.

✎ *Pencil Problem #6* ✐

6. Solve by factoring.

6a. $5x^2 = 20x$

6b. $x^2 = 8x - 15$

Objective #7: Solve quadratic equations by the square root property.

✔ *Solved Problem #7*

7. Solve by the square root property.

7a. $3x^2 - 21 = 0$

$$3x^2 - 21 = 0$$
$$3x^2 = 21$$
$$x^2 = 7$$
$$x = \pm\sqrt{7}$$

The solution set is $\left\{-\sqrt{7}, \sqrt{7}\right\}$.

✎ *Pencil Problem #7* ✐

7. Solve by the square root property.

7a. $5x^2 + 1 = 51$

7b. $(x+5)^2 = 11$

$$(x+5)^2 = 11$$
$$x+5 = \pm\sqrt{11}$$
$$x = -5 \pm \sqrt{11}$$

The solution set is $\{-5+\sqrt{11},\ -5-\sqrt{11}\}$.

7b. $3(x-4)^2 = 15$

Objective #8: Solve quadratic equations by completing the square.

✔ *Solved Problem #8*

8. Solve by completing the square: $x^2 + 4x - 1 = 0$

$$x^2 + 4x - 1 = 0$$
$$x^2 + 4x \quad = 1$$

Half of 4 is 2, and 2^2 is 4, which should be added to both sides.

$$x^2 + 4x + 4 = 1 + 4$$
$$x^2 + 4x + 4 = 5$$
$$(x+2)^2 = 5$$
$$x+2 = \sqrt{5} \quad \text{or} \quad x+2 = -\sqrt{5}$$
$$x = -2+\sqrt{5} \qquad x = -2-\sqrt{5}$$

The solution set is $\left\{-2 \pm \sqrt{5}\right\}$.

✎ *Pencil Problem #8* ✎

8. Solve by completing the square: $x^2 + 6x - 11 = 0$

Objective #9: Solve quadratic equations using the quadratic formula.

✔ *Solved Problem #9*

9. Solve using the quadratic formula: $2x^2 + 2x - 1 = 0$

The equation is in the form $ax^2 + bx + c = 0,$ where $a = 2$, $b = 2$, and $c = -1$.

$$
\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)} \\
&= \frac{-2 \pm \sqrt{4+8}}{4} \\
&= \frac{-2 \pm \sqrt{12}}{4} \\
&= \frac{-2 \pm 2\sqrt{3}}{4} \\
&= \frac{2(-1 \pm \sqrt{3})}{4} \\
&= \frac{-1 \pm \sqrt{3}}{2}
\end{aligned}
$$

The solution set is $\left\{\frac{-1+\sqrt{3}}{2}, \frac{-1-\sqrt{3}}{2}\right\}$.

✎ *Pencil Problem #9* ✎

9. Solve using the quadratic formula: $3x^2 - 3x - 4 = 0$

Objective #10: Use the discriminant to determine the number and type of solutions.

✔ *Solved Problem #10*

10. Compute the discriminant and determine the number and type of solutions: $3x^2 - 2x + 5 = 0$

$$
\begin{aligned}
b^2 - 4ac &= (-2)^2 - 4(3)(-5) \\
&= -56
\end{aligned}
$$

Since the discriminant is negative, there is no real solution.

✎ *Pencil Problem #10* ✎

10. Compute the discriminant and determine the number and type of solutions: $2x^2 - 11x + 3 = 0$

Objective #11: Determine the most efficient method to use when solving a quadratic equation.

✔ *Solved Problem #11*

11. What is the most efficient method for solving a quadratic equation of the form $ax^2 + c = 0$?

The most efficient method is to solve for x^2 and apply the square root property.

Pencil Problem #11

11. What is the most efficient method for solving a quadratic equation of the form $u^2 = d$, where u is a first-degree polynomial?

Objective #12: Solve radical equations.

✔ *Solved Problem #12*

12. Solve: $\sqrt{x+3}+3=x$

$$\sqrt{x+3}+3=x$$
$$\sqrt{x+3}=x-3$$
$$(\sqrt{x+3})^2=(x-3)^2$$
$$x+3=x^2-6x+9$$
$$0=x^2-7x+6$$
$$0=(x-6)(x-1)$$
$$x-6=0 \quad \text{or} \quad x-1=0$$
$$x=6 \qquad\qquad x=1$$

Check 6: $\sqrt{x+3}+3=x$

$$\sqrt{6+3}+3=6$$
$$6=6$$

Check 1: $\sqrt{x+3}+3=x$

$$\sqrt{1+3}+3=1$$
$$5=1$$

The solution set is $\{6\}$.

Pencil Problem #12

12. Solve: $\sqrt{2x+13}=x+7$

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1. $\{9\}$ *(P.7 #9)*

2. $\left\{\frac{33}{2}\right\}$ *(P.7 #11)*

3a. $\{-3\}$ *(P.7 #23)* **3b.** $\varnothing$ *(P.7 #25)*

4. $f=\frac{pq}{p+q}$ *(P.7 #39)*

5. $\left\{-\frac{5}{3},\ 3\right\}$ *(P.7 #47)*

6a. $\{0, 4\}$ *(P.7 #59)* **6b.** $\{3, 5\}$ *(P.7 #57)*

7a. $\{-\sqrt{10},\ \sqrt{10}\}$ *(P.7 #63)* **7b.** $\{4+\sqrt{5},\ 4-\sqrt{5}\}$ *(P.7 #65)*

8. $\{3+\sqrt{11},\ 3-\sqrt{11}\}$ *(P.7 #71)*

9. $\left\{\frac{3+\sqrt{57}}{6},\ \frac{3-\sqrt{57}}{6}\right\}$ *(P.7 #79)*

10. 97; two unequal real solutions *(P.7 #85)*

11. The square root property *(P.7 #101)*

12. $\{-6\}$ *(P.7 #119)*

Section P.8
Modeling with Equations

Counting Your Money!

From how much you can expect to earn at your first job after college to how much you need to save each month for retirement, mathematical models can help you plan your finances.
In this section, you will see applications that involve starting salaries with a college degree based on major, discounts on electronics, and sharing the cost of a yacht.

Objective #1: Use equations to solve problems.

✔ *Solved Problem #1*

1a. The median starting salary of a computer science major exceeds that of an education major by $21 thousand. The median starting salary of an economics major exceeds that of an education major by $14 thousand. Combined their median starting salaries are $140 thousand. Determine the median starting salary for each of these three majors.

Since the salaries of both the computer science and economics majors are compared to education majors, we let x = the median starting salary for an education major.

The other two majors have salaries that exceed this salary by a specified amount, so we add that amount to the salary for the education major.

$x + 21$ = the median starting salary for a computer science major

$x + 14$ = the median starting salary for an economics major

Since the combined salary is $140 thousand, we add the three salaries and set the sum equal to 140. Then we solve for x.

$$\begin{aligned} x+(x+21)+(x+14) &= 140 \\ 3x+35 &= 140 \\ 3x &= 105 \\ x &= 35 \end{aligned}$$

$x + 21 = 35 + 21 = 56$
$x + 14 = 35 + 14 = 49$

The median starting salaries are $35 thousand for an education major, $56 thousand for a computer science major, and $49 thousand for an economics major.

You should verify that these salaries are $140 thousand combined.

✎ *Pencil Problem #1* ✎

1a. According to the American Bureau of Labor Statistics, you will devote 37 years to sleeping and watching TV. The number of years sleeping will exceed the number of years watching TV by 19. Over your lifetime, how many years will you spend on each of these activities?

1b. After a 30% price reduction, you purchase a new computer for \$840. What was the computer's price before the reduction?

Let x = the computer's price before the reduction.

Since the price reduction is 30% of the original price, the discount can be expressed as $0.30x$. The price is reduced by this amount, so the reduced price can be found by subtracting the discount from the original price: $x - 0.30x$.

The reduced price is \$840, so $x - 0.30x = 840$.

$$\begin{aligned} x - 0.30x &= 840 \\ 1x - 0.30x &= 840 \\ (1 - 0.30)x &= 840 \\ 0.70x &= 840 \\ \frac{0.70x}{0.70} &= \frac{840}{0.70} \\ x &= 1200 \end{aligned}$$

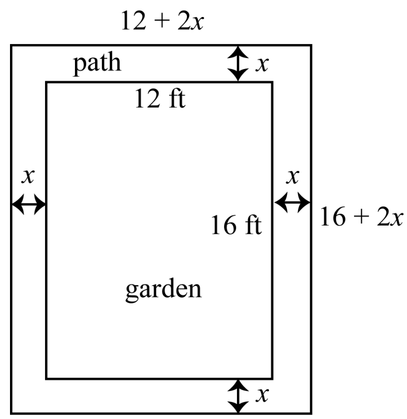
Before the reduction, the computer's price was \$1200.

1b. After a 20% reduction, you purchase a television for \$336. What was the television's price before the reduction?

1c. A rectangular garden measures 16 feet by 12 feet. A path of uniform width is to be added so as to surround the entire garden. The landscape artist doing the work wants the garden and path to cover an area of 320 square feet. How wide should the path be?

Let x = the width of the path.

Since the path is added on all sides of the garden, the length of the larger rectangle including the garden and the path is $16 + 2x$, and the width is $12 + 2x$. See the figure. Since the area of the larger rectangle is to be 320 square feet, we write the equation using $A = lw$.



$$\begin{aligned}(16+2x)(12+2x) &= 320\\ 192+56x+4x^2 &= 320\\ 4x^2+56x-128 &= 0\\ 4(x^2+14x-32) &= 0\\ 4(x+16)(x-2) &= 0\end{aligned}$$

$$x+16=0 \quad \text{or} \quad x-2=0$$
$$x=-16 \qquad\qquad x=2$$

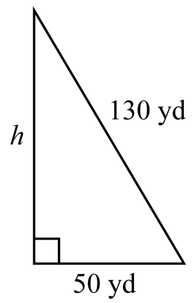
Since the width of the path cannot be negative, discard −16.

If the path is 2 feet wide, then the length of the larger rectangle is 20 feet and the width is 16 feet, resulting in an area of 320 square feet. The width of the path is 2 feet.

1c. A pool measuring 10 meters by 20 meters is surrounded by a path of uniform width. If the area of the pool and the path combined is 600 square meters, what is the width of the path?

1d. A radio tower is supported by two wires that are each 130 yards long and attached to the ground 50 yards from the base of the tower. How tall is the tower?

Let h = the height of the tower. The situation is illustrated by the right triangle in the figure.



We write an equation using the Pythagorean Theorem, where the lengths of the legs are h and 50 and the length of the hypotenuse is 130.

$$h^2 + 50^2 = 130^2$$

Solve the equation using the square root property.

$$h^2 + 50^2 = 130^2$$
$$h^2 + 2500 = 16,900$$
$$h^2 = 14,400$$
$$h = \pm\sqrt{14,400} = \pm 120$$

Disregard the negative value. You should check that a height of 120 yards satisfies the conditions of the problem. The radio tower is 120 yards tall.

1d. A 20-foot ladder is 15 feet from a house. How far up the house, to the nearest tenth of a foot, does the ladder reach?

✔ *Solved Problem #1e*

1e. A group of people equally share in a \$5,000,000 lottery. Before the money is divided, three more winning ticket holders are declared. As a result, each person's share is reduced by \$375,000. How many people were in the original group of winners?

Let x = the number of people in the original group.

Then $x + 3$ = the number of people in the new group.

The share for each person in the original group, $\frac{5{,}000{,}000}{x}$, reduced by 375,000 is the share for each person in the new group, $\frac{5{,}000{,}000}{x+3}$. The equation is $\frac{5{,}000{,}000}{x} - 375{,}000 = \frac{5{,}000{,}000}{x+3}$.

The LCD is $x(x + 3)$.

$$\frac{5{,}000{,}000}{x} - 375{,}000 = \frac{5{,}000{,}000}{x+3}$$

$$x(x+3)\left(\frac{5{,}000{,}000}{x} - 375{,}000\right) = x(x+3)\cdot\frac{5{,}000{,}000}{x+3}$$

$$\cancel{x}(x+3)\cdot\frac{5{,}000{,}000}{\cancel{x}} - x(x+3)\cdot 375{,}000 = x\cancel{(x+3)}\cdot\frac{5{,}000{,}000}{\cancel{x+3}}$$

$$5{,}000{,}000(x+3) - 375{,}000x(x+3) = 5{,}000{,}000x$$

$$5{,}000{,}000x + 15{,}000{,}000 - 375{,}000x^2 - 1{,}125{,}000x = 5{,}000{,}000x$$

$$-375{,}000x^2 - 1{,}125{,}000x + 15{,}000{,}000 = 0$$

$$-375{,}000(x^2 + 3x - 40) = 0$$

$$-375{,}000(x+8)(x-5) = 0$$

$$x + 8 = 0 \quad \text{or} \quad x - 5 = 0$$

$$x = -8 \qquad\qquad x = 5$$

Disregard the negative value. If 5 people share \$5,000,000, each person receives \$1,000,000. If 3 more people join the group, then each receives \$625,000, which is \$375,000 less than \$1,000,000.
There were 5 people in the original group.

✎ *Pencil Problem #1e* ✐

1e. A group of people equally share in a $20,000,000 lottery. Before the money is divided, two more winning ticket holders are declared. As a result, each person's share is reduced by $500,000. How many people were in the original group of winners?

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1a. TV: 9 years; sleeping: 28 years *(P.8 #1)*
1b. $420 *(P.8 #11)*
1c. 5 meters *(P.8 #27)*
1d. 13.2 feet *(P.8 #31)*
1e. 8 people *(P.8 #37)*

Section P.9
Linear Inequalities and Absolute Values Inequalities

Are You in LOVE?

As the years go by in a relationship, three key components of love…

passion

commitment

intimacy

…progress differently over time.

Passion peaks early in a relationship and then declines.
By contrast, intimacy and commitment build gradually.

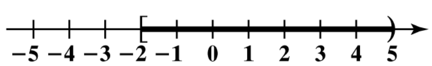
In the applications of this section of the textbook, we will use mathematics to explore the relationships among these three variables of love.

Objective #1: Use interval notation.

✔ *Solved Problem #1*

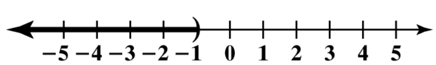
1a. Express $[-2,5)$ in set-builder notation and graph.

$\{x|-2 \le x < 5\}$



1b. Express $(-\infty,-1)$ in set-builder notation and graph.

$\{x|x < -1\}$



Pencil Problem #1

1a. Express $(1,6]$ in set-builder notation and graph.

1b. Express $[-3,\infty)$ in set-builder notation and graph.

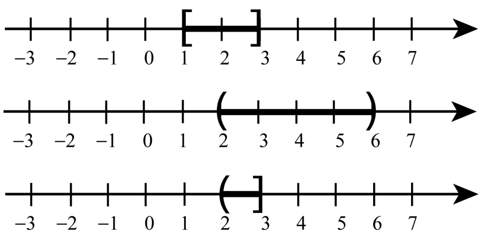
Objective #2: Find intersections and unions of intervals.

✔ *Solved Problem #2*

2. Use graphs to find each set:

2a. $[1, 3] \cap (2, 6)$

Graph each interval. The intersection consists of the portion of the number line that the two graphs have in common.



$[1, 3] \cap (2, 6) = (2, 3]$

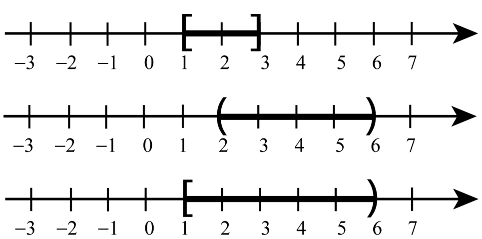
✎ *Pencil Problem #2* ✐

2. Use graphs to find each set:

2a. $(-3, 0) \cap [-1, 2]$

2b. $[1, 3] \cup (2, 6)$

Graph each interval. The union consists of the portion of the number line in either one of the intervals or the other or both.



$[1, 3] \cup (2, 6) = [1, 6)$

2b. $(-3, 0) \cup [-1, 2]$

Objective #3: Solve linear inequalities.

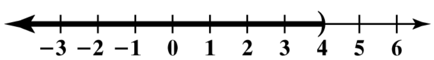
✔ Solved Problem #3

3a. Solve and graph the solution set on a number line:

$$3x+1>7x-15$$

$$\begin{aligned} 3x+1&>7x-15 \\ -4x&>-16 \\ \frac{-4x}{-4}&<\frac{-16}{-4} \\ x&<4 \end{aligned}$$

The solution set is $(-\infty,4)$.

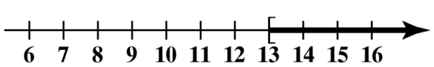


3b. Solve and graph the solution set on a number line:

$$\frac{x-4}{2}\ge\frac{x-2}{3}+\frac{5}{6}$$

$$\begin{aligned} \frac{x-4}{2}&\ge\frac{x-2}{3}+\frac{5}{6} \\ 6\left(\frac{x-4}{2}\right)&\ge6\left(\frac{x-2}{3}+\frac{5}{6}\right) \\ 3(x-4)&\ge2(x-2)+5 \\ 3x-12&\ge2x-4+5 \\ 3x-12&\ge2x+1 \\ x&\ge13 \end{aligned}$$

The solution set is $[13,\infty)$.



✎ Pencil Problem #2 ✎

3a. Solve and graph the solution set on a number line:

$$-9x\ge36$$

3b. Solve and graph the solution set on a number line:

$$\frac{x}{4}-\frac{3}{2}=\frac{x}{2}+1$$

3c. A car can be rented from Basic Rental for $260 per week with no extra charge for mileage. Continental charges $80 per week plus 25 cents for each mile driven to rent the same car. How many miles must be driven in a week to make the rental cost for Basic Rental a better deal than Continental's?

Let $x =$ number of miles driven in a week.

$$\overbrace{260}^{\text{Cost for Basic Rental}} < \overbrace{80+0.25x}^{\text{Cost for Continental}}$$

$$260 < 80+0.25x$$
$$180 < 0.25x$$
$$\frac{180}{0.25} < \frac{0.25x}{0.25}$$
$$720 < x$$
$$x > 720$$

Driving more than 720 miles per week makes Basic Rental a better deal.

3c. An elevator at a construction site has a maximum capacity of 3000 pounds. If the elevator operator weighs 245 pounds and each cement bag weighs 95 pounds, how many bags of cement can be safely lifted on the elevator in one trip?

Objective #4: Solve compound inequalities.

✔ *Solved Problem #4*

4. Solve the compound inequality: $1 \le 2x+3 < 11$

$$1 \le 2x+3 < 11$$
$$1-3 \le 2x+3-3 < 11-3$$
$$-2 \le 2x < 8$$
$$\frac{-2}{2} \le \frac{2x}{2} < \frac{8}{2}$$
$$-1 \le x < 4$$

The solution set is $[-1, 4)$.

✎ *Pencil Problem #4* ✐

4. Solve the compound inequality: $-11 < 2x-1 \le -5$

Objective #5: Solve absolute value inequalities.

✔ *Solved Problem #5*

5a. Solve the inequality:

$$|x-2|<5$$

Rewrite without absolute value bars.
$|u|<c$ means $-c<u<c$.

$$-5<x-2<5$$
$$-5+2<x-2+2<5+2$$
$$-3<x<7$$

The solution set is $(-3,7)$.

5b. Solve the inequality:

$$-3|5x-2|+20\geq -19$$

First, isolate the absolute value expression on one side of the inequality.

$$-3|5x-2|+20\geq -19$$
$$-3|5x-2|\geq -39$$
$$\frac{-3|5x-2|}{-3}\leq\frac{-39}{-3}$$
$$|5x-2|\leq 13$$

Rewrite $|5x-2|\leq 13$ without absolute value bars.
$|u|\leq c$ means $-c\leq u\leq c$.

$$-13\leq 5x-2\leq 13$$
$$-13+2\leq 5x-2+2\leq 13+2$$
$$-11\leq 5x\leq 15$$
$$\frac{-11}{5}\leq\frac{5x}{5}\leq\frac{15}{5}$$
$$-\frac{11}{5}\leq x\leq 3$$

The solution set is $\left[\frac{-11}{5},3\right]$.

✎ *Pencil Problem #5* ✎

5a. Solve the inequality:

$$|2x-6|<8$$

5b. Solve the inequality:

$$|2(x-1)+4|\leq 8$$

5c. Solve the inequality: $18 < |6 - 3x|$

Rewrite with the absolute value expression on the left.
$|6 - 3x| > 18$
This means the same as $6 - 3x < -18$ or $6 - 3x > 18$.

$$\begin{array}{lcl} 6 - 3x < -18 & \text{or} & 6 - 3x > 18 \\ -3x < -24 & & -3x > 12 \\ \dfrac{-3x}{-3} > \dfrac{-24}{-3} & & \dfrac{-3x}{-3} < \dfrac{12}{-3} \\ x > 8 & & x < -4 \end{array}$$

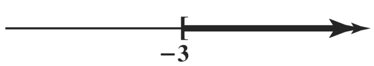
The solution set is $\{x \mid x < -4 \text{ or } x > 8\}$ or
$(-\infty, -4) \cup (8, \infty)$.

5c. Solve the inequality: $1 < |2 - 3x|$

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

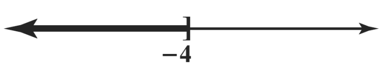
1a. $\{x \mid 1 < x \leq 6\}$; 

(P.9 #1)

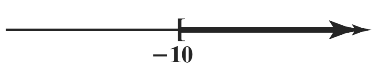
1b. $\{x \mid x \geq -3\}$; 

(P.9 #9)

2a. $[-1, 0)$ *(P.9 #15)* **2b.** $(-3, 2]$ *(P.9 #17)*

3a. $(-\infty, -4]$; 

(P.9 #31)

3b. $[-10, \infty)$; 

(P.9 #41) **3c.** at most 29 bags *(P.9 #129)*

4. $(-5, -2]$ *(P.9 #55)*

5a. $(-1, 7)$ *(P.9 #65)* **5b.** $[-5, 3]$ *(P.9 #63)* **5c.** $\left(-\infty, \frac{1}{3}\right) \cup (1, \infty)$ *(P.9 #87)*