Section 9.1
The Ellipse

Do You Trust Politicians?

The U.S. Capitol Building is beautiful both inside, and out. But did you know that part of its architecture includes an elliptical ceiling in Sanctuary Hall?

John Quincy Adams, while a member of the house of Representatives, discovered that he could use the reflective properties of the room, which we will study in this section, to eavesdrop on the conversations of other House members.

Objective #1: Graph ellipses centered at the origin.

✔ *Solved Problem #1*

1. Graph and locate the foci: $16x^2+9y^2=144$

First, write the equation in standard form.

$$16x^2+9y^2=144$$

$$\frac{16x^2}{144}+\frac{9y^2}{144}=\frac{144}{144}$$

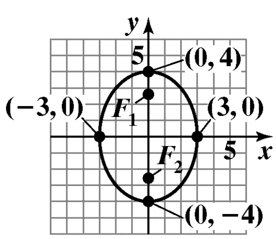
$$\frac{x^2}{9}+\frac{y^2}{16}=1$$

Because the denominator of the y^2-term is greater than the denominator of the x^2-term, the major axis is vertical.

Since $a^2=16$, $a=4$ and the vertices are $(0,-4)$ and $(0,4)$.

Since $b^2=9$, $b=3$ and endpoints of the minor axis are $(-3,0)$ and $(3,0)$.

$c^2=a^2-b^2=16-9=7$, $c=\sqrt{7}$ and the foci are $(0,-\sqrt{7})$ and $(0,\sqrt{7})$.



$16x^2 + 9y^2 = 144$

✎ *Pencil Problem #1*

1. Graph and locate the foci: $\frac{x^2}{16}+\frac{y^2}{4}=1$

Objective #2: Write equations of ellipses in standard form.

✔ *Solved Problem #2*

2. Find the standard form of the equation of an ellipse with foci at (–2, 0) and (2, 0) and vertices (–3, 0) and (3, 0).

Because the foci are located on the *x*-axis, the major axis is horizontal with the center midway between them at (0, 0). The form of the equation is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. We need to determine values for a^2 and b^2. The distance from the center to either vertex is 3, so $a = 3$ and $a^2 = 9$. The distance from the center to either focus is 2, so $c = 2$.

$$b^2 = a^2 - c^2 = 3^2 - 2^2 = 5$$

The equation is $\frac{x^2}{9}+\frac{y^2}{5}=1$.

✎ *Pencil Problem #2* ✎

2. Find the standard form of the equation of an ellipse with foci at (0, –4) and (0, 4) and vertices (0, –7) and (0, 7).

Objective #3: Graph ellipses not centered at the origin.

✔ *Solved Problem #3*

3. Graph: $\frac{(x+1)^2}{9}+\frac{(y-2)^2}{4}=1$. Where are the foci located?

$$\frac{(x+1)^2}{9}+\frac{(y-2)^2}{4}=1$$

The center of the ellipse is $(-1,2)$.

Because the denominator of the x^2-term is greater than the denominator of the y^2-term, the major axis is horizontal.

Since $a^2=9$, $a=3$ and the vertices lie 3 units to the right and left of the center.

Since $b^2=4$, $b=2$ and endpoints of the minor axis lie 2 units above and below the center.

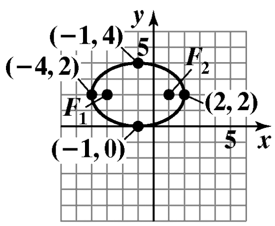
Since $c^2=a^2-b^2=9-4=5$, $c=\sqrt{5}$ and the foci are located $\sqrt{5}$ units to the right and left of center.

The following chart summarizes these key points.

✎ *Pencil Problem #3* ✎

3. Graph: $\frac{(x-4)^2}{9}+\frac{(y+2)^2}{25}=1$. Where are the foci located?

Center	Vertices	Endpoints Minor Axis	Foci
$(-1,2)$	$(-1-3,2)$ $=(-4,2)$	$(-1,2-2)$ $=(-1,0)$	$(-1-\sqrt{5},2)$
	$(-1+3,2)$ $=(2,2)$	$(-1,2+2)$ $=(-1,4)$	$(-1+\sqrt{5},2)$



$$\frac{(x+1)^2}{9}+\frac{(y-2)^2}{4}=1$$

Objective #4: Solve applied problems involving ellipses.

✔ *Solved Problem #4*

4. A semielliptical archway over a one-way road has a height of 10 feet and a width of 40 feet. Your truck has a width of 12 feet and a height of 9 feet. Will your truck clear the opening of the archway?

Using the equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ the archway can be expressed as $\frac{x^2}{20^2}+\frac{y^2}{10^2}=1$ or $\frac{x^2}{400}+\frac{y^2}{100}=1$.

Since the truck is 12 feet wide, we need to determine the height of the archway at $\frac{12}{2}=6$ feet from the center.

Substitute 6 for x to find the height y.

$$\frac{x^2}{400}+\frac{y^2}{100}=1$$
$$\frac{6^2}{400}+\frac{y^2}{100}=1$$

✎ *Pencil Problem #4* ✎

4. Will a truck that is 8 feet wide carrying a load that reaches 7 feet above the ground clear the semielliptical arch on the one-way road that passes under a bridge that has a height of 10 feet and a width of 30 feet?

Solve for y.

$$\frac{6^2}{400}+\frac{y^2}{100}=1$$

$$\frac{36}{400}+\frac{y^2}{100}=1$$

$$400\left(\frac{36}{400}+\frac{y^2}{100}\right)=400(1)$$

$$36+4y^2=400$$

$$4y^2=364$$

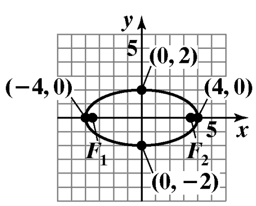
$$y^2=91$$

$$y=\sqrt{91}\approx 9.54$$

The height of the archway 6 feet from the center is approximately 9.54 feet.

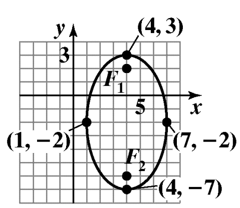
Since the truck is 9 feet high, the truck will clear the archway.

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:



1. $\frac{x^2}{16}+\frac{y^2}{4}=1$ foci at $(-2\sqrt{3},0)$ and $(2\sqrt{3},0)$ *(9.1 #1)*

2. $\frac{x^2}{33}+\frac{y^2}{49}=1$ *(9.1 #27)*



3. $\frac{(x-4)^2}{9}+\frac{(y+2)^2}{25}=1$ foci at $(4, 2)$ and $(4,-6)$ *(9.1 #41)*

4. Yes; the height of the archway 4 feet from the center is approximately 9.64 feet. *(9.1 #65)*

Section 9.2
The Hyperbola

Sonic Boom !

When a jet flies at a speed greater than the speed of sound, the shock wave that is created is heard as a sonic boom.

The wave has the shape of a cone.
The shape formed as the cone hits the ground is one branch of a hyperbola, the topic of this section of the textbook.

Objective #1: Locate a hyperbola's vertices and foci.

✔ *Solved Problem #1*

1a. Find the vertices and locate the foci for the hyperbola with the given equation: $\frac{x^2}{25}-\frac{y^2}{16}=1.$

The x^2 – term is positive.
Therefore, the transverse axis lies along the *x*–axis.

Since $a^2=25$ and $a=5,$ the vertices are $(-5,0)$ and $(5,0).$

Since $c^2=a^2+b^2=25+16=41,$ $c=\sqrt{41}$ and the foci are $(-\sqrt{41},0)$ and $(\sqrt{41},0)$.

1b. Find the vertices and locate the foci for the hyperbola with the given equation: $\frac{y^2}{25}-\frac{x^2}{16}=1.$

The y^2 – term is positive.
Therefore, the transverse axis lies along the *y*–axis.

Since $a^2=25$ and $a=5,$ the vertices are $(0,-5)$ and $(0,5).$

Since $c^2=a^2+b^2=25+16=41,$ $c=\sqrt{41}$ and the foci are $(0,-\sqrt{41})$ and $(0,\sqrt{41})$.

✎ *Pencil Problem #1* ✎

1a. Find the vertices and locate the foci for the hyperbola with the given equation: $\frac{x^2}{4}-\frac{y^2}{1}=1.$

1b. Find the vertices and locate the foci for the hyperbola with the given equation: $\frac{y^2}{4}-\frac{x^2}{1}=1.$

Objective #2: Write equations of hyperbolas in standard form.

✔ *Solved Problem #2*

2. Find the standard form of the equation of a hyperbola with foci at (0, –5) and (0, 5) and vertices (0, –3) and (0, 3).

Because the foci are located on the *y*-axis, the transverse axis lies on the *y*-axis with the center midway between the foci at (0, 0). The form of the equation is $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$. We need to determine values for a^2 and b^2.

The distance from the center to either vertex is 3, so $a = 3$ and $a^2 = 9$. The distance from the center to either focus is 5, so $c = 5$.

$b^2 = c^2 - a^2 = 5^2 - 3^2 = 16$

The equation is $\frac{y^2}{9}-\frac{x^2}{16}=1$.

✎ *Pencil Problem #2* ✎

2. Find the standard form of the equation of a hyperbola with foci at (–4, 0) and (4, 0) and vertices (–3, 0) and (3, 0).

Objective #3: Graph hyperbolas centered at the origin.

✔ *Solved Problem #3*

3a. Graph and locate the foci: $\frac{x^2}{36}-\frac{y^2}{9}=1$. What are the equations of the asymptotes?

$$\frac{x^2}{36}-\frac{y^2}{9}=1$$

Since the x^2 – term is positive, the transverse axis lies along the *x*–axis.

Since $a^2 = 36$ and $a = 6$, the vertices are $(-6,0)$ and $(6,0)$.

Construct a rectangle using –6 and 6 on the *x*–axis, and –3 and 3 on the *y*–axis.

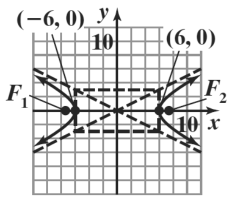
Draw extended diagonals to obtain the asymptotes.

The equations of the asymptotes are $y=\pm\frac{3}{6}x=\pm\frac{1}{2}x$.

✎ *Pencil Problem #3* ✎

3a. Graph and locate the foci: $\frac{x^2}{9}-\frac{y^2}{25}=1$. What are the equations of the asymptotes?

Draw the two branches of the hyperbola by starting at each vertex and approaching the asymptotes.



$\frac{x^2}{36} - \frac{y^2}{9} = 1$

Since $c^2 = a^2 + b^2 = 36 + 9 = 45$, $c = \sqrt{45} = 3\sqrt{5}$ and the foci are located at $(-3\sqrt{5}, 0)$ and $(3\sqrt{5}, 0)$.

3b. Graph and locate the foci: $y^2 - 4x^2 = 4$. What are the equations of the asymptotes?

First write the equation in standard form.

$$y^2 - 4x^2 = 4$$

$$\frac{y^2}{4} - \frac{4x^2}{4} = \frac{4}{4}$$

$$\frac{y^2}{4} - \frac{x^2}{1} = 1$$

The equation is in the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ with $a^2 = 4$ and $b^2 = 1$.

The transverse axis lies on the y-axis and the vertices are $(0, -2)$ and $(0, 2)$.

Because $a^2 = 4$ and $b^2 = 1$, $a = 2$ and $b = 1$.

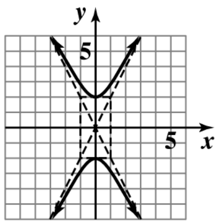
Construct a rectangle using -2 and 2 on the y–axis, and -1 and 1 on the x–axis.

Draw extended diagonals to obtain the asymptotes.

The equations of the asymptotes are $y = \pm\frac{2}{1}x = \pm 2x$.

3b. Graph and locate the foci: $9y^2 - 25x^2 = 225$. What are the equations of the asymptotes?

Draw the two branches of the hyperbola by starting at each vertex and approaching the asymptotes.



$y^2 - 4x^2 = 4$

Since $c^2 = a^2 + b^2 = 4 + 1 = 5,\ c = \sqrt{5}$ and the foci are located at $(0, -\sqrt{5})$ and $(0, \sqrt{5})$.

Objective #4: Graph hyperbolas not centered at the origin.

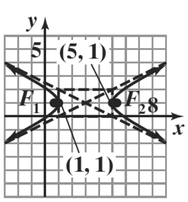
✔ *Solved Problem #4*

4. Graph: $\frac{(x-3)^2}{4} - \frac{(y-1)^2}{1} = 1.$ Where are the foci located? What are the equations of the asymptotes?

Because the term involving x^2 has the positive coefficient, the transverse axis is horizontal. Based on the standard form $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$, we see that $h = 3$ and $k = 1$ so the center is (3, 1). We also see that $a^2 = 4$ and $b^2 = 1$, so $a = 2$ and $b = 1$.

Since $a = 2$, the vertices are 2 units to the left and right of the center at (3 − 2, 1), or (1, 1), and (3 + 2, 1), or (5, 1). Draw a rectangle using the vertices, (1, 1) and (5, 1) and the points $b = 1$ unit above and below the center. Draw the extended diagonals to obtain the asymptotes.
The asymptotes of the unshifted hyperbola are $y = \pm\frac{b}{a}x = \pm\frac{1}{2}x.$ Thus, the asymptotes of the shifted hyperbola are $y - 1 = \pm\frac{1}{2}(x-3).$

Draw the two branches of each hyperbola by starting at each vertex and approaching the asymptotes.



$\frac{(x-3)^2}{4} - \frac{(y-1)^2}{1} = 1$

Since $c^2 = a^2 + b^2 = 4 + 1 = 5,\ c = \sqrt{5}$ and the foci are located $\sqrt{5}$ units to the left and right of center at $(3-\sqrt{5},\ 1)$ and $(3+\sqrt{5},\ 1)$.

✎ *Pencil Problem #4* ✐

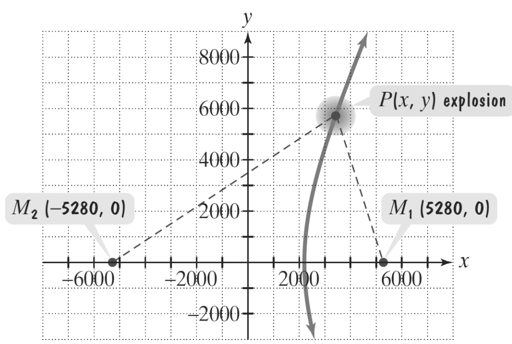
4. Graph: $\frac{(y+2)^2}{4} - \frac{(x-1)^2}{16} = 1.$ Where are the foci located? What are the equations of the asymptotes?

Objective #5: Solve applied problems involving hyperbolas.

✔ Solved Problem #5

5. An explosion is recorded by two microphones that are 2 miles apart. Microphone M_1 received the sound 3 seconds before microphone M_2. Assuming sound travels at 1100 feet per second, determine the possible locations of the explosion relative to the location of the microphones.

Because 1 mile = 5280 feet, place microphone M_1 at (5280, 0) in a coordinate system. Since the microphones are two miles apart, place M_2 at (–5280, 0). Assume that the explosion is at point $P(x, y)$ in the coordinate system. The set of all possible points for the explosion is a hyperbola with the microphones at the foci.



Since M_1 received the sound 3 seconds before microphone M_2 and sound travels at 1100 feet per second, the difference between the distances from P to M_1 and from P to M_2 is 3300 feet. Thus,
$2a = 3300$ and $a = 1650$, so $a^2 = 2{,}722{,}500$.

The distance from the center, (0, 0), to either focus is 5280, so $c = 5280$.

$$b^2 = c^2 - a^2 = 5280^2 - 1650^2 = 25{,}155{,}900$$

The equation of the hyperbola is
$\frac{x^2}{2{,}722{,}500} - \frac{y^2}{25{,}155{,}900} = 1.$ The explosion occurred somewhere on the right branch of this hyperbola (the branch closer to M_1).

✎ Pencil Problem #5 ✎

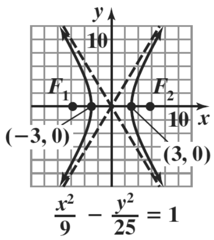
5. An explosion is recorded by two microphones that are 1 mile apart. Microphone M_1 received the sound 2 seconds before microphone M_2. Assuming sound travels at 1100 feet per second, determine the possible locations of the explosion relative to the location of the microphones.

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

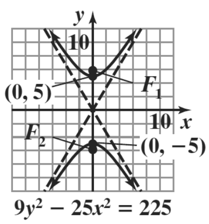
1a. vertices: $(-2,0)$ and $(2,0)$; foci: $(-\sqrt{5},0)$ and $(\sqrt{5},0)$ *(9.2 #1)*

1b. vertices: $(0,-2)$ and $(0,2)$; foci: $(0,-\sqrt{5})$ and $(0,\sqrt{5})$ *(9.2 #3)*

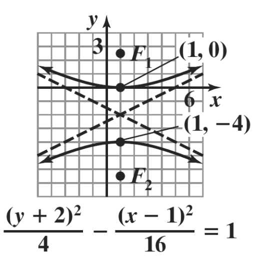
2. $\dfrac{x^2}{9}-\dfrac{x^2}{7}=1$ *(9.2 #7)*



3a. asymptotes: $y=\pm\dfrac{5}{3}x$; foci: $(-\sqrt{34},0)$ and $(\sqrt{34},0)$ *(9.2 #13)*



3b. asymptotes: $y=\pm\dfrac{5}{3}x$; foci: $(0,-\sqrt{34})$ and $(0,\sqrt{34})$ *(9.2 #23)*



4. asymptotes: $y+2=\pm\dfrac{1}{2}(x-1)$; foci: $(1,-2-2\sqrt{5})$ and $(1,-2+2\sqrt{5})$ *(9.2 #37)*

5. If M_1 is located 2640 feet to the right of the origin on the *x*-axis, the explosion is located on the right branch of the hyperbola given by the equation $\dfrac{x^2}{1{,}210{,}000}-\dfrac{x^2}{5{,}759{,}600}=1$. *(9.2 #61)*

Section 9.3
The Parabola

How Good Is Your Reception?

In this section we study parabolas and their properties.
A satellite dish is in the shape of a parabolic surface.
Signals coming from a satellite strike the surface of the dish and are reflected to the focus, where the receiver is located.

The applications in the Exercise Set include concepts from each of the conic sections that we have studied.

Objective #1: Graph parabolas with vertices at the origin.

✔ *Solved Problem #1*

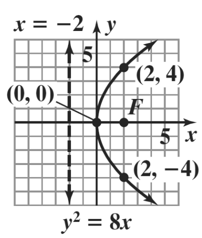
1a. Find the focus and directrix of the parabola given by $y^2 = 8x$. Then graph the parabola.

The equation $y^2 = 8x$ is in the standard form $y^2 = 4px$, so $4p = 8$ and $p = 2$. Because p is positive, the parabola opens to the right. The focus is 2 units to the right of the vertex, (0, 0), at $(p, 0)$ or (2, 0). The directrix is 2 units to the left of the vertex: $x = -p$ or $x = -2$.

To graph the parabola, substitute 2 for x in the equation.

$$y^2 = 8 \cdot 2$$
$$y^2 = 16$$
$$y = \pm\sqrt{16} = \pm 4$$

The points (2, 4) and (2, −4) are on the parabola above and below the focus.



✎ *Pencil Problem #1* ✎

1a. Find the focus and directrix of the parabola given by $y^2 = 16x$. Then graph the parabola.

1b. Find the focus and directrix of the parabola given by $x^2 = -12y.$ Then graph the parabola.

The equation $x^2 = -12y$ is in the standard form $x^2 = 4py,$ so $4p = -12$ and $p = -3$. Because p is negative, the parabola opens downward. The focus is 3 units below the vertex, (0, 0), at (0, p) or (0, –3). The directrix is 3 units above the vertex: $y = -p$ or $y = 3$.

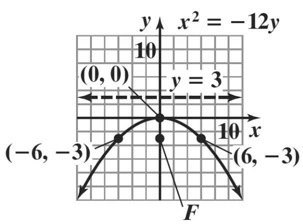
To graph the parabola, substitute –3 for y in the equation.

$$x^2 = -12(-3)$$

$$x^2 = 36$$

$$x = \pm\sqrt{36} = \pm 6$$

The points (–6, –3) and (6, –3) are on the parabola to the left and right of the focus.



1b. Find the focus and directrix of the parabola given by $x^2 = -16y.$ Then graph the parabola.

Objective #2: Write equations of parabolas in standard form.

✔ *Solved Problem #2*

2. Find the standard form of the equation of a parabola with focus (8, 0) and directrix $x = -8$.

The vertex of the parabola is midway between the focus and the directrix at (0, 0). Since the focus is on the x-axis, we use the standard form $y^2 = 4px.$

The focus is 8 units to the right of the vertex, so $p = 8$.

The equation is $y^2 = 4 \cdot 8x$ or $y^2 = 32x.$

✎ *Pencil Problem #2* ✐

2. Find the standard form of the equation of a parabola with focus (0, 15) and directrix $y = -15$.

Objective #3: Graph parabolas with vertices not at the origin.

✔ *Solved Problem #3*

3a. Find the vertex, focus, and directrix of the parabola given by $(x-2)^2 = 4(y+1).$ Then graph the parabola.

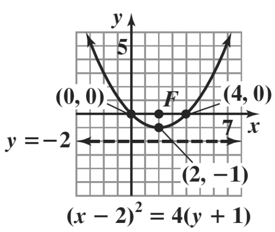
Writing the equation as $(x-2)^2 = 4(y-(-1)),$ we see that $h = 2$ and $k = -1$. The vertex is $(h, k) = (2, -1)$.

✎ *Pencil Problem #3* ✐

3a. Find the vertex, focus, and directrix of the parabola given by $(x+1)^2 = -8(y+1).$ Then graph the parabola.

Because $4p = 4$, $p = 1$. The focus is 1 unit above the vertex at $(h, k + p) = (2, -1 + 1) = (2, 0)$. The directrix is 1 unit below the vertex: $y = k - p = -1 - 1$ or $y = -2$.

The length of the latus rectum is $|4p| = |4 \cdot 1| = |4| = 4$. The latus rectum extends 2 units to the left and right of the focus. The endpoints of the latus rectum are $(2 - 2, 0)$ or $(0, 0)$ and $(2 + 2, 0)$ or $(4, 0)$.



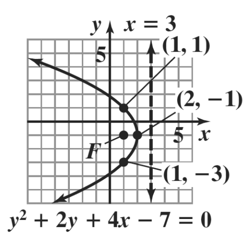
3b. Find the vertex, focus, and directrix of the parabola given by $y^2 + 2y + 4x - 7 = 0$. Then graph the parabola.

Complete the square on y.

$$y^2 + 2y + 4x - 7 = 0$$
$$y^2 + 2y = -4x + 7$$
$$y^2 + 2y + 1 = -4x + 8$$
$$(y+1)^2 = -4(x-2)$$

We see that $k = -1$ and $h = 2$, so the vertex is at $(h, k) = (2, -1)$. Since $4p = -4$, $p = -1$. The focus is 1 unit to the left of the vertex at $(h, k) = (2 - 1, -1) = (1, -1)$. The directrix is 1 unit to the right of the vertex: $x = h - p = 2 - (-1)$ or $x = 3$.

The length of the latus rectum is $|4p| = |4(-1)| = |-4| = 4$. The latus rectum extends 2 units above and below the focus. The endpoints of the latus rectum are $(1, -1 + 2)$ or $(1, 1)$ and $(1, -1 - 2)$ or $(1, -3)$.



3b. Find the vertex, focus, and directrix of the parabola given by $y^2 - 2y + 12x - 35 = 0$. Then graph the parabola.

Objective #4: Solve applied problems involving parabolas.

✔ *Solved Problem #2*

4. An engineer is designing a flashlight using a parabolic mirror and a light source. The casting has a diameter of 6 inches and a depth of 4 inches. What is the equation of the parabola used to shape the mirror? At what point should the light source be placed relative to the mirror's vertex?

Position the parabola with its vertex at the origin and opening upward. Then the focus is at (0, p) on the y-axis. Since the casting has a diameter of 6 inches, it extends 3 units to the left and right of the y-axis. Since it is 4 inches deep, the point (3, 4) is on the parabola.

We use the standard form $x^2 = 4py$. Using (3, 4), we have

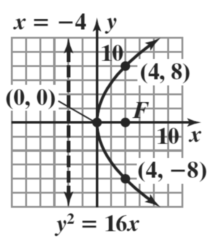
$$3^2 = 4p \cdot 4$$
$$9 = 16p$$
$$\frac{9}{16} = p.$$

Thus, the equation is $x^2 = 4 \cdot \frac{9}{16} y$ or $x^2 = \frac{9}{4} y$. The light source should be placed at the focus, $(0, p) = (0, \frac{9}{16})$, or $\frac{9}{16}$ inch above the vertex along the axis of symmetry.

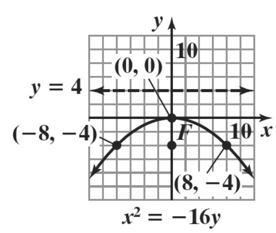
Pencil Problem #2

4. The reflector of a flashlight is in the shape of a parabolic surface. The casting has a diameter of 4 inches and a depth of 1 inch. What is the equation of the parabola used to shape the mirror? At what point should the light source be placed relative to the mirror's vertex?

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

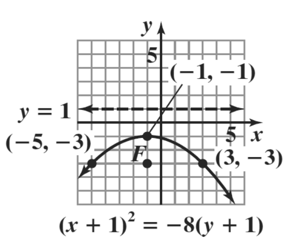


1a. focus: (4, 0); directrix: $x = -4$ *(9.3 #5)*

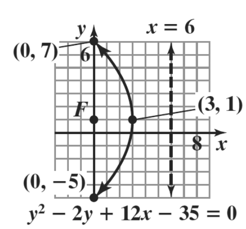


1b. focus: (0, −4); directrix: $y = 4$ *(9.3 #11)*

2. $x^2 = 60y$ *(9.3 #21)*



3a. vertex: (−1, −1); focus: (−1, −3); directrix: $y = 1$ *(9.3 #37)*



3b. vertex: (3, 1); focus: (0, 1); directrix: $x = 6$ *(9.3 #45)*

4. $x^2 = 4y$; 1 inch above the vertex along the axis of symmetry *(9.3 #61)*

Section 9.4
Rotation of Axes

A New Twist on Conics

Well, technically it's a rotation not a twist, but you get the idea.
Not all ellipses, hyperbolas, and parabolas have axes that are parallel to the x- or y-axis. When an axis of a conic section is not parallel to the x- or y-axis, we will introduce a new rectangular coordinate system with an x'-axis and a y'-axis that intersect at the origin of the original system. The technique is called a rotation of axes. In the rotated system, the conic will have a form that we've already studied.

Objective #1: Identify conics without completing the square.

✔ ***Solved Problem #1***

1a. Identify the graph of $3x^2 + 2y^2 + 12x - 4y + 2 = 0$.

The coefficient of x^2 is 3: $A = 3$.
The coefficient of y^2 is 2: $C = 2$.
$AC = 3(2) = 6$

Since $A \neq C$ and $AC > 0$, the graph of the equation is an ellipse.

1b. Identify the graph of $x^2 + y^2 - 6x + y + 3 = 0$.

The coefficient of x^2 is 1: $A = 1$.
The coefficient of y^2 is 1: $C = 1$.

Since $A = C$, the graph of the equation is a circle.

1c. Identify the graph of $y^2 - 12x - 4y + 52 = 0$.

There is no x^2-term, so the coefficient of x^2 is 0: $A = 0$.
The coefficient of y^2 is 1: $C = 1$.
$AC = 0(1) = 0$

Since $AC = 0$, the graph of the equation is a parabola.

1d. Identify the graph of $9x^2 - 16y^2 - 90x + 64y + 17 = 0$.

The coefficient of x^2 is 9: $A = 9$.
The coefficient of y^2 is −16: $C = -16$.
$AC = 9(-16) = -144$

Since $A \neq C$ and $AC < 0$, the graph of the equation is a hyperbola.

✎ *Pencil Problem #1* ✐

1a. Identify the graph of $9x^2 + 4y^2 - 36x + 8y + 31 = 0$.

1b. Identify the graph of $4x^2 + 4y^2 + 12x + 4y + 1 = 0$.

1c. Identify the graph of $y^2 - 4x + 2y + 21 = 0$.

1d. Identify the graph of $100x^2 - 7y^2 + 90y - 368 = 0$.

Objective #2: Use rotation of axes formulas.

✔ Solved Problem #2

2. Write the equation $xy = 2$ in terms of a rotated $x'y'$-system if the angle of rotation from the x-axis to the x'-axis is 45°. Express the equation in standard form.

With $\theta = 45°$, the rotation formulas are

$$\begin{aligned} x &= x'\cos\theta - y'\sin\theta \\ &= x'\cos 45° - y'\sin 45° \\ &= x'\left(\frac{\sqrt{2}}{2}\right) - y'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(x' - y') \end{aligned}$$

$$\begin{aligned} y &= x'\sin\theta + y'\cos\theta \\ &= x'\sin 45° + y'\cos 45° \\ &= x'\left(\frac{\sqrt{2}}{2}\right) + y'\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}(x' + y') \end{aligned}$$

Now substitute into the equation $xy = 2$. Multiply and simplify.

$$\begin{aligned} xy &= 2 \\ \left[\frac{\sqrt{2}}{2}(x' - y')\right]\left[\frac{\sqrt{2}}{2}(x' + y')\right] &= 2 \\ \frac{2}{4}(x' - y')(x' + y') &= 2 \\ \frac{1}{2}(x'^2 - y'^2) &= 2 \\ \frac{1}{4}(x'^2 - y'^2) &= 1 \\ \frac{x'^2}{4} - \frac{y'^2}{4} &= 1 \end{aligned}$$

This is the standard form of the equation of a hyperbola.

✎ Pencil Problem #2 ✎

2. Write the equation $xy = -1$ in terms of a rotated $x'y'$-system if the angle of rotation from the x-axis to the x'-axis is 45°. Express the equation in standard form.

Objective #3: Write equations of rotated conics in standard form.

✔ *Solved Problem #3a*

3a. Rewrite the equation $2x^2+\sqrt{3}xy+y^2-2=0$ in a rotated $x'y'$-system without an $x'y'$-term. Express the equation in the standard form of a conic section. Graph the conic section in the rotated system.

Find $\cot 2\theta$.
The coefficient of x^2 is 2: $A = 2$.
The coefficient of xy is $\sqrt{3}$: $B=\sqrt{3}$.
The coefficient of y^2 is 1: $C = 1$.

$$\cot 2\theta=\frac{A-C}{B}=\frac{2-1}{\sqrt{3}}=\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}}{3}$$

Find the angle of rotation θ.

Since $\cot 60^\circ=\frac{\sqrt{3}}{3}$, we have $2\theta=60^\circ$. Thus, $\theta=30^\circ$.

Substitute $\theta=30^\circ$ into the rotation formulas.

$$x=x'\cos\theta-y'\sin\theta=x'\cos 30^\circ-y'\sin 30^\circ=x'\left(\frac{\sqrt{3}}{2}\right)-y'\left(\frac{1}{2}\right)=\frac{\sqrt{3}x'-y'}{2}$$

$$y=x'\sin\theta+y'\cos\theta=x'\sin 30^\circ+y'\cos 30^\circ=x'\left(\frac{1}{2}\right)+y'\left(\frac{\sqrt{3}}{2}\right)=\frac{x'+\sqrt{3}y'}{2}$$

Now substitute into the equation $2x^2+\sqrt{3}xy+y^2-2=0$.

$$2x^2+\sqrt{3}xy+y^2-2=0$$

$$2\left(\frac{\sqrt{3}x'-y'}{2}\right)^2+\sqrt{3}\left(\frac{\sqrt{3}x'-y'}{2}\right)\left(\frac{x'+\sqrt{3}y'}{2}\right)+\left(\frac{x'+\sqrt{3}y'}{2}\right)^2-2=0$$

$$2\left(\frac{3x'^2-2\sqrt{3}x'y'+y'^2}{4}\right)+\sqrt{3}\left(\frac{\sqrt{3}x'^2+3x'y'-x'y'-\sqrt{3}y'^2}{4}\right)+\left(\frac{x'^2+2\sqrt{3}x'y'+3y'^2}{4}\right)-2=0$$

$$2(3x'^2-2\sqrt{3}x'y'+y'^2)+\sqrt{3}(\sqrt{3}x'^2+2x'y'-\sqrt{3}y'^2)+(x'^2+2\sqrt{3}x'y'+3y'^2)-8=0$$

$$6x'^2-4\sqrt{3}x'y'+2y'^2+3x'^2+2\sqrt{3}x'y'-3y'^2+x'^2+2\sqrt{3}x'y'+3y'^2-8=0$$

$$10x'^2+2y'^2-8=0$$

Write the equation in standard form.

$$10x'^2+2y'^2-8=0$$

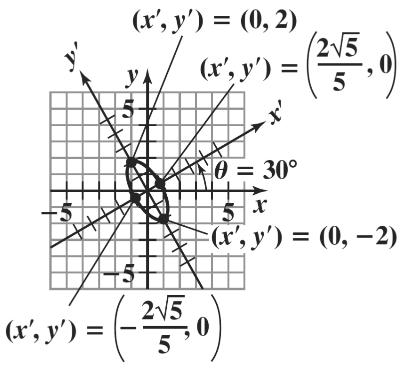
$$10x'^2+2y'^2=8$$

$$\frac{10x'^2}{8}+\frac{2y'^2}{8}=1$$

$$\frac{5x'^2}{4}+\frac{y'^2}{4}=1$$

$$\frac{x'^2}{\frac{4}{5}}+\frac{y'^2}{4}=1$$

This is the equation of an ellipse with vertices at (0, 2) and (0, −2) on the y'-axis. The endpoints of the minor axis are at $\left(\frac{2\sqrt{5}}{5},\ 0\right)$ and $\left(-\frac{2\sqrt{5}}{5},\ 0\right)$.



✎ *Pencil Problem #3a* ✎

3a. Rewrite the equation $11x^2 + 10\sqrt{3}xy + y^2 - 4 = 0$ in a rotated $x'y'$-system without an $x'y'$-term. Express the equation in the standard form of a conic section. Graph the conic section in the rotated system.

✔ *Solved Problem #3b*

3b. Rewrite the equation $4x^2-4xy+y^2-8\sqrt{5}x-16\sqrt{5}y=0$ in a rotated $x'y'$-system without an $x'y'$-term. Express the equation in the standard form of a conic section. Graph the conic section in the rotated system.

Find $\cot 2\theta$.
The coefficient of x^2 is 4: $A = 4$.
The coefficient of xy is −4: $B = -4$.
The coefficient of y^2 is 1: $C = 1$.

$$\cot 2\theta=\frac{A-C}{B}=\frac{4-1}{-4}=-\frac{3}{4}$$

Use $\cot 2\theta=-\frac{3}{4}$ to find $\sin\theta$ and $\cos\theta$. Begin by finding $\cos 2\theta$.

Since $\cot 2\theta$ is negative, 2θ is a quadrant II angle. In quadrant II, x is negative and y is positive.

$$\cot 2\theta=-\frac{3}{4}=\frac{x}{y}=\frac{-3}{4}$$

We let $x = -3$ and $y = 4$. Then $r=\sqrt{(-3)^2+4^2}=\sqrt{9+16}=\sqrt{25}=5$. Use x and r to find $\cos 2\theta$.

$$\cos 2\theta=\frac{x}{r}=\frac{-3}{5}=-\frac{3}{5}$$

Now use identities to find $\sin\theta$ and $\cos\theta$. Since 2θ is a quadrant II angle, θ is a quadrant I angle. Sine and cosine are both positive in quadrant I.

$$\sin\theta=\sqrt{\frac{1-\cos 2\theta}{2}}=\sqrt{\frac{1-\left(-\frac{3}{5}\right)}{2}}=\sqrt{\frac{\frac{5}{5}+\frac{3}{5}}{2}}=\sqrt{\frac{\frac{8}{5}}{2}}=\sqrt{\frac{4}{5}}=\frac{2}{\sqrt{5}}=\frac{2\sqrt{5}}{5}$$

$$\cos\theta=\sqrt{\frac{1+\cos 2\theta}{2}}=\sqrt{\frac{1+\left(-\frac{3}{5}\right)}{2}}=\sqrt{\frac{\frac{5}{5}-\frac{3}{5}}{2}}=\sqrt{\frac{\frac{2}{5}}{2}}=\sqrt{\frac{1}{5}}=\frac{1}{\sqrt{5}}=\frac{\sqrt{5}}{5}$$

Substitute $\sin\theta=\frac{2\sqrt{5}}{5}$ and $\cos\theta=\frac{\sqrt{5}}{5}$ into the rotation formulas.

$$x=x'\cos\theta-y'\sin\theta=x'\left(\frac{\sqrt{5}}{5}\right)-y'\left(\frac{2\sqrt{5}}{5}\right)=\frac{\sqrt{5}}{5}(x'-2y')$$

$$y=x'\sin\theta+y'\cos\theta=x'\left(\frac{2\sqrt{5}}{5}\right)+y'\left(\frac{\sqrt{5}}{5}\right)=\frac{\sqrt{5}}{5}(2x'+y')$$

Now substitute the expressions for x and y into the equation $4x^2-4xy+y^2-8\sqrt{5}x-16\sqrt{5}y=0$.

$$4\left[\frac{\sqrt{5}}{5}(x'-2y')\right]^2-4\left[\frac{\sqrt{5}}{5}(x'-2y')\right]\left[\frac{\sqrt{5}}{5}(2x'+y')\right]+\left[\frac{\sqrt{5}}{5}(2x'+y')\right]^2$$
$$-8\sqrt{5}\left[\frac{\sqrt{5}}{5}(x'-2y')\right]-16\sqrt{5}\left[\frac{\sqrt{5}}{5}(2x'+y')\right]=0$$

(continued on next page)

Begin simplifying. Square and distribute as appropriate. Note that $\left(\frac{\sqrt{5}}{5}\right)^2 = \frac{(\sqrt{5})^2}{5^2} = \frac{5}{25} = \frac{1}{5}$ and

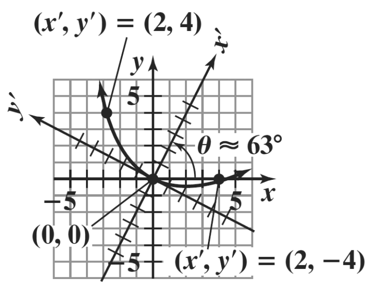
$\sqrt{5} \cdot \frac{\sqrt{5}}{5} = \frac{\sqrt{5} \cdot \sqrt{5}}{5} = \frac{5}{5} = 1.$

$$4 \cdot \frac{1}{5}(x'^2 - 4x'y' + 4y'^2) - 4 \cdot \frac{1}{5}(2x'^2 - 3x'y' - 2y'^2) + \frac{1}{5}(4x'^2 + 4x'y' + y'^2)$$
$$-8x' + 16y' - 32x' - 16y' = 0$$

Distribute again and begin combining like terms. (Note that you could multiply by 5 to eliminate fractions.) The coefficients of the terms involving x'^2, $x'y'$, and y' each have a sum of 0.

$$\frac{4}{5}x'^2 - \frac{16}{5}x'y' + \frac{16}{5}y'^2 - \frac{8}{5}x'^2 + \frac{12}{5}x'y' + \frac{8}{5}y'^2 + \frac{4}{5}x'^2 + \frac{4}{5}x'y' + \frac{1}{5}y'^2 - 40x' = 0$$
$$\frac{16}{5}y'^2 + \frac{8}{5}y'^2 + \frac{1}{5}y'^2 - 40x' = 0$$
$$\frac{25}{5}y'^2 - 40x' = 0$$
$$5y'^2 = 40x'$$
$$y'^2 = 8x'$$

This is the standard form of the equation of parabola with vertex at (0, 0). Letting $4p = 8$, we see that $p = 2$. The focus is at (2, 0). Substituting 2 for x', we get $y'^2 = 16$, so $y' = \pm 4$. The endpoints of the latus rectum are (2, 4) and (−2, 4). By solving $\cos\theta = \frac{\sqrt{5}}{5}$ for θ, we have $\theta = \cos^{-1}\frac{\sqrt{5}}{5} \approx 63°$.



✎ Pencil Problem #3b ✎

3b. Rewrite the equation $34x^2 - 24xy + 41y^2 - 25 = 0$ in a rotated $x'y'$-system without an $x'y'$-term. Express the equation in the standard form of a conic section. Graph the conic section in the rotated system.

Objective #4: Identify conics without rotating axes.

✔ ***Solved Problem #4***

4. Identify the graph of $3x^2 - 2\sqrt{3}xy + y^2 + 2x + 2\sqrt{3}y = 0.$

The coefficient of x^2 is 3: $A = 3$.
The coefficient of xy is $-2\sqrt{3}$: $B = -2\sqrt{3}$.
The coefficient of y^2 is 1: $C = 1$.
$B^2 - 4AC = (-2\sqrt{3})^2 - 4(3)(1) = 12 - 12 = 0$

Since $B^2 - 4AC = 0,$ the graph of the equation is a parabola.

✎ ***Pencil Problem #4*** ✎

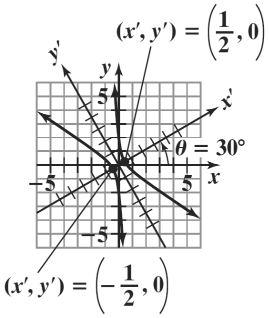
4. Identify the graph of $5x^2 - 2xy + 5y^2 - 12 = 0.$

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1a. ellipse *(9.4 #6)* **1b.** circle *(9.4 #5)* **1c.** parabola *(9.4 #1)* **1d.** hyperbola *(9.4 #7)*

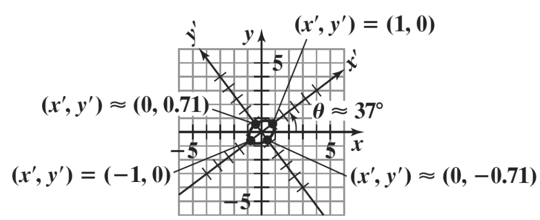
2. $\frac{y'^2}{2} - \frac{x'^2}{2} = 1$ *(9.4 #9)*

3a. $\frac{x'^2}{\frac{1}{4}} - \frac{y'^2}{1} = 1$



(9.4 #31)

3b. $\frac{x'^2}{1} + \frac{y'^2}{\frac{1}{2}} = 1$



(9.4 #37)

4. ellipse or circle *(9.4 #39)*

Section 9.5
Parametric Equations

Home Run!

The ball is hit. You watch as it flies through the air. As the distance from the batter increases, the height of the ball above the ground increases until it reaches its highest point and then decreases. Will it make it? Yes, home run!

In the Exercise Set, you will see how the motion of the baseball can be described by a pair of equations, one representing the distance of the ball from the batter and the other representing it height above the ground. Such equations, called *parametric equations*, allow us to describe both the path of an object and its location on that path at a particular time.

Objective #1: Use point plotting to graph plane curves described by parametric equations.

✔ *Solved Problem #1*

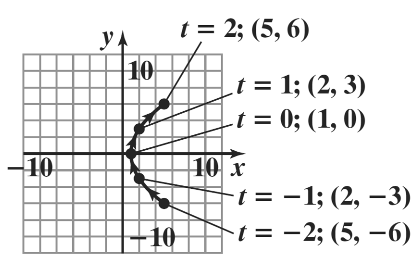
1. Graph the plane curve defined by the parametric equations

$x = t^2 + 1, \quad y = 3t, \quad -2 \le t \le 2.$

We will select integer values for t on the given interval. Let $t = -2, -1, 0, 1,$ and 2. Calculate values of x and y for each value of t and form the ordered pairs (x, y).

t	$x = t^2 + 1$	$y = 3t$	(x, y)
-2	$(-2)^2 + 1 = 5$	$3(-2) = -6$	$(5, -6)$
-1	$(-1)^2 + 1 = 2$	$3(-1) = -3$	$(2, -3)$
0	$0^2 + 1 = 1$	$3(0) = 0$	$(1, 0)$
1	$1^2 + 1 = 2$	$3(1) = 3$	$(2, 3)$
2	$2^2 + 1 = 5$	$3(2) = 6$	$(5, 6)$

Plot the points in order of increasing values of t and connect them with a smooth curve. Arrows along the graph indicate the direction, or orientation, of the curve as t increases from -2 to 2.



✎ *Pencil Problem #1* ✎

1. Graph the plane curve defined by the parametric equations

$x = t + 2, \quad y = t^2, \quad -2 \le t \le 2.$

Objective #2: Eliminate the parameter.

✔ *Solved Problem #2*

2a. Sketch the plane curve represented by the parametric equations $x = \sqrt{t}$ and $y = 2t - 1$ by eliminating the parameter.

We begin by solving for t in one of the equations. Solve $x = \sqrt{t}$ for t by squaring both sides.

$x = \sqrt{t}$

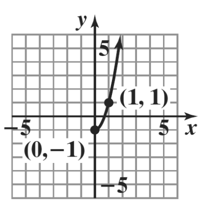
$x^2 = t,\ x \geq 0$

In the original equation, x is the principal square of t, which is never negative, so $x \geq 0$. Now substitute x^2 for t in $y = 2t - 1$.

$y = 2t - 1$

$y = 2x^2 - 1$

Graph $y = 2x^2 - 1$ on the restricted domain $x \geq 0$.



2b. Sketch the plane curve represented by the parametric equations $x = 6\cos t$, $y = 4\sin t$, $\pi \leq t \leq 2\pi$ by eliminating the parameter.

We will begin by rewriting both equations in anticipation of applying the identity $\sin^2 t + \cos^2 t = 1$. Solve each equation for the trigonometric function and then square each side.

$x = 6\cos t$	$y = 4\sin t$
$\dfrac{x}{6} = \cos t$	$\dfrac{y}{4} = \sin t$
$\left(\dfrac{x}{6}\right)^2 = (\cos t)^2$	$\left(\dfrac{y}{4}\right)^2 = (\sin t)^2$
$\dfrac{x^2}{36} = \cos^2 t$	$\dfrac{y^2}{16} = \sin^2 t$

✎ *Pencil Problem #2* ✎

2a. Sketch the plane curve represented by the parametric equations $x = \sqrt{t}$ and $y = t - 1$ by eliminating the parameter.

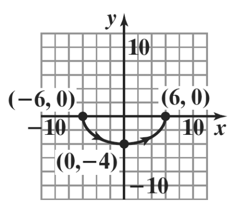
2b. Sketch the plane curve represented by the parametric equations $x = 2\sin t$, $y = 2\cos t$, $0 \leq t \leq 2\pi$ by eliminating the parameter.

Now replace $\cos^2 t$ with $\frac{x^2}{36}$ and $\sin^2 t$ with $\frac{y^2}{16}$ in the identity $\sin^2 t + \cos^2 t = 1$.

$$\sin^2 t + \cos^2 t = 1$$

$$\frac{y^2}{16} + \frac{x^2}{36} = 1 \text{ or } \frac{x^2}{36} + \frac{y^2}{16} = 1$$

This is the equation of an ellipse with vertices at (−6, 0) and (6, 0) and endpoints of the minor axis at (−4, 0) and (4, 0). However, in the original equations, we had a restriction on t: $\pi \le t \le 2\pi$. On this interval, $y = 4\sin t$ is always negative or 0, so $y \le 0$. The graph is the lower half of the ellipse just described.



Objective #3: Find parametric equations for functions.

✔ *Solved Problem #3*

3. Find parametric equations for the parabola whose equation is $y = x^2 - 25$.

Let $x = t$. Then parametric equations for $y = x^2 - 25$ are $x = t$ and $y = t^2 - 25$.

✎ *Pencil Problem #3* ✎

3. Find parametric equations for the parabola whose equation is $y = x^2 + 4$.

Objective #4: Understand the advantages of parametric representations.

✔ *Solved Problem #4*

4. True or false: Parametric equations provide more information than rectangular equations because they can describe both the path of a moving object in the plane in terms of coordinates (x, y) and the time the object is at each point.

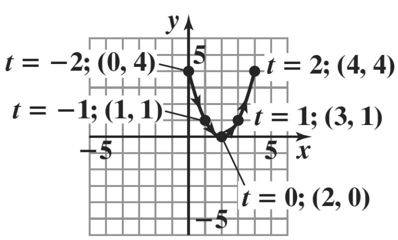
True; the parameter t in the equations that define x and y often represents time. Without, the parameter, we can describe the path of an object but not the time it is at each point along the path.

✎ *Pencil Problem #4* ✐

4. True or false: When using a graphing utility, you must eliminate the parameter before you can graph parametric equations.

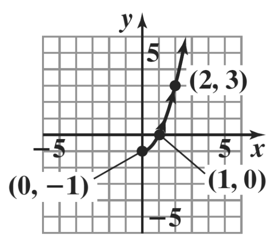
Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1.



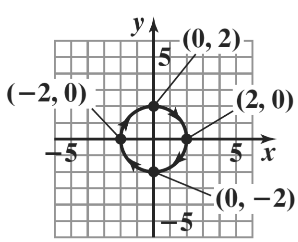
(9.5 #9)

2a. $y = x^2 - 1,\ x \geq 0$



(9.5 #25)

2b. $\frac{x^2}{4} + \frac{y^2}{4} = 1$



(9.5 #27)

3. $x = t$ and $y = t^2 + 4$ *(9.5 #55)*

4. false

Section 9.6
Conic Sections in Polar Coordinates

Back to the Drawing Board!

You know how ellipses and hyperbolas are defined in terms of distances from fixed points in the plane using rectangular coordinates. In this section, we want to study the conic sections in polar coordinates but not by converting their equations by algebraic means. Instead, we start with new, but equivalent, definitions of the conics.

Objective #1: Define conics in terms of a focus and a directrix.

✔ *Solved Problem #1*

1. True or false: If the eccentricity of a conic is $e = 1$, then the conic is an ellipse.

False; when the eccentricity of a conic is 1, the distance from a point on the conic to the focus is equal to its distance from the directrix: When $e = \frac{PF}{PD} = 1,\ PF = PD.$ This describes a parabola.

✎ *Pencil Problem #1* ✐

1. True or false: If the eccentricity of a conic is $e = 4$, then the conic is an ellipse.

Objective #2: Graph the polar equations of conics.

✔ *Solved Problem #2*

2a. Graph the polar equation: $r = \frac{4}{2 - \cos\theta}.$

Notice that the constant term in the denominator is not 1, so the equation is not in standard form. Divide the numerator and the denominator by 2, so that the constant will be 1.

$$r = \frac{\frac{4}{2}}{\frac{2}{2} - \frac{\cos\theta}{2}} = \frac{2}{1 - \frac{1}{2}\cos\theta}$$

The equation is now in the standard form $r = \frac{ep}{1 - e\cos\theta},$ where $ep = 2$ and $e = \frac{1}{2}.$ Using these facts, $ep = \frac{1}{2}p = 2,$ so $p = 4.$ Because $e = \frac{1}{2} < 1,$ the conic is an ellipse.

✎ *Pencil Problem #2* ✐

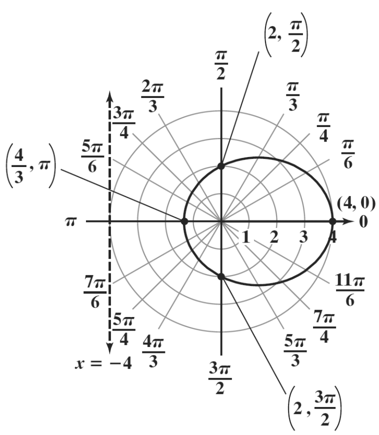
2a. Graph the polar equation: $r = \frac{12}{5 + 3\cos\theta}.$

For equations in the form $r = \frac{ep}{1 - e\cos\theta}$, one focus is at the pole, the directrix is the line $x = -p$ or $x = -4$, located 4 units to the left of the pole, and the graph has polar axis symmetry.

The major axis of the ellipse is on the polar axis, so let $\theta = 0$ and $\theta = \pi$ to find the vertices. The vertices are (4, 0) and $\left(\frac{4}{3}, \pi\right)$. Pick some other values of θ between 0 and π to sketch the upper half of the ellipse.

θ	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$
r	$\frac{8}{3}$	2	$\frac{8}{5}$

Reflect the upper half about the polar axis to complete the ellipse.



2b. Graph the polar equation: $r = \frac{8}{4 + 4\sin\theta}$.

Notice that the constant term in the denominator is not 1, so the equation is not in standard form. Divide the numerator and the denominator by 4, so that the constant will be 1.

$$r = \frac{\frac{8}{4}}{\frac{4}{4} + \frac{4\sin\theta}{4}} = \frac{2}{1 + \sin\theta}$$

2b. Graph the polar equation: $r = \frac{6}{2 - 2\sin\theta}$.

The equation is now in the standard form $r = \dfrac{ep}{1 + e\sin\theta}$, where $ep = 2$ and $e = 1$. Using these facts, $ep = 1p = 2$, so $p = 2$. Because $e = 1$, the conic is a parabola.

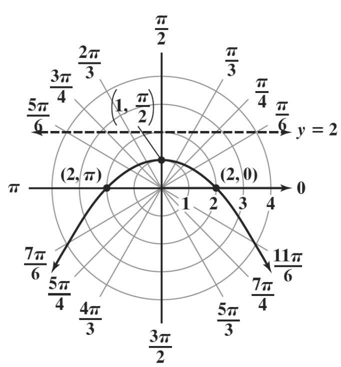
For equations in the form $r = \dfrac{ep}{1 + e\sin\theta}$, one focus is at the pole, the directrix is the line $y = p$ or $y = 2$, located 2 units above the pole, and the graph has symmetry with respect to the line $\theta = \dfrac{\pi}{2}$.

Let $\theta = \dfrac{\pi}{2}$ to find the vertex; the vertex is $\left(1, \dfrac{\pi}{2}\right)$.

Pick some other values of θ between $-\dfrac{\pi}{2}$ and $\dfrac{\pi}{2}$ to sketch the right half of the parabola.

θ	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$
r	4	2	$\frac{4}{3}$

Reflect the right half about the line $\theta = \dfrac{\pi}{2}$ to complete the parabola.



2c. Graph the polar equation: $r = \dfrac{9}{3 - 9\cos\theta}$.

Notice that the constant term in the denominator is not 1, so the equation is not in standard form. Divide the numerator and the denominator by 3, so that the constant will be 1.

$$r = \frac{\frac{9}{3}}{\frac{3}{3} - \frac{9}{3}\cos\theta} = \frac{3}{1 - 3\cos\theta}$$

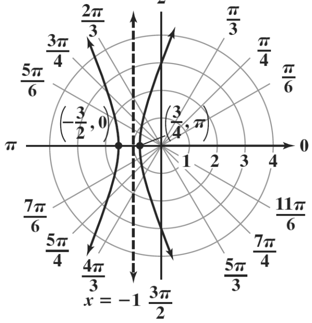
The equation is now in the standard form $r = \dfrac{ep}{1 - e\cos\theta}$, where $ep = 3$ and $e = 3$. Using these facts, $ep = 3p = 3$, so $p = 1$. Because $e = 3 > 1$, the conic is a hyperbola.

For equations in the form $r = \dfrac{ep}{1 - e\cos\theta}$, one focus is at the pole, the directrix is the line $x = -p$ or $x = -1$, located 1 unit to the left of the pole, and the graph has polar axis symmetry.

The transverse axis of the hyperbola is on the polar axis, so let $\theta = 0$ and $\theta = \pi$ to find the vertices. The vertices are $\left(-\frac{3}{2},\ 0\right)$ and $\left(\frac{3}{4},\ \pi\right)$. Pick some other values of θ to sketch the upper right portion of the hyperbola.

θ	$\frac{\pi}{2}$	$\frac{2\pi}{3}$
r	3	$\frac{6}{5}$

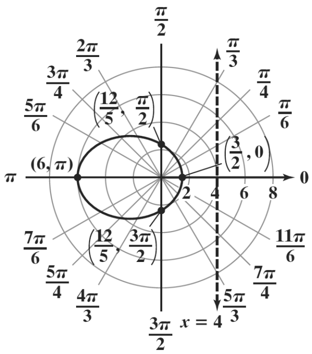
Reflect the upper right portion about the polar axis to complete the right half of the hyperbola. Draw the mirror image of the right half through the other vertex to complete the graph.



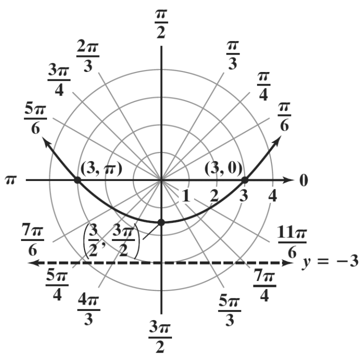
2c. Graph the polar equation: $r = \dfrac{8}{2 - 4\cos\theta}$.

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

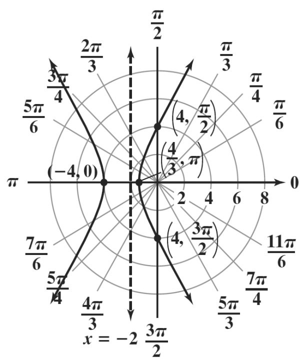
1. False



2a. . *(9.6 #13)*



2b. *(9.6 #15)*



2c. *(9.6 #17)*